

# Economics Education and Research Consortium



## ENTRANCE EXAMINATION IN MATHEMATICS

*Surname* \_\_\_\_\_

*Name* \_\_\_\_\_

1	2	3	4	5	6	7	8	Total

Tbilisi, June 2007

1. Random variable  $\xi$  has the following probability density function:

$$p(x) = \begin{cases} C(1-x^2), & x \in [-1;1], \\ 0, & x \notin [-1;1]. \end{cases}$$

Determine:

1.1. Coefficient  $C$ , (1 point)

1.2. Distribution Function  $F_{\xi}(x)$ , (2 points)

1.3. Mathematical Expectation  $E\xi$ , (1 point)

1.4. Dispersion  $D\xi$ , (2 points)

1.5. Mode  $Mo\xi$  (1 point)

1.6. Median  $Md\xi$ . (1 point)

2. Find an inverse matrix to the given:

$$A = \begin{pmatrix} 2 & -3 & 1 \\ -2 & 7 & 2 \\ 3 & 2 & 4 \end{pmatrix} \quad (2 \text{ points})$$

3. For which value of parameter  $x$  the following vectors

$$\vec{f}_1 = \begin{pmatrix} x^2 \\ 3 \\ 1 \end{pmatrix} \quad \vec{f}_2 = \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix} \quad \vec{f}_3 = \begin{pmatrix} -1 \\ 5 \\ -4 \end{pmatrix} \quad \text{will be coplanar vectors? (2 points)}$$

4. Two players **A** and **B** roll the pair of ordinary dice in turn. Player **A** rolls first. To win, a player must show a sum of dots on the two dice equal to nine. Game is going on until somebody wins. Compute the probability for the second player **B** to win. (3 points)

5. An employment agency claims that it finds jobs for 85% of the people who use its services. A newspaper reporter investigating the claims of the agency did a survey of people who had used the agency. The results showed that of the 90 people polled only 70 had found jobs through the agency. Find the probability that 70 or fewer have found jobs through the agency if the agency's 85% placement success claim is true. (3 points)

6. Solve the given Cauchy problem: 
$$\begin{cases} y' + y \cdot \operatorname{tg} x = \frac{1}{\cos x} . \\ y(0) = 0 \end{cases} \quad (3 \text{ points})$$

7. Find the maximum and minimum values of the function

$$f(x) = \frac{1}{9}x^3 - \frac{1}{6}x^2 - \frac{2}{3}x + 1 \quad \text{over the interval } [-3, 3]. \quad (2 \text{ points})$$

8. For the curve given by  $e^{xy^2} - 2x - 4y = 1$  find the slope of and equation for the tangent line at the point  $(x, y) = (0, 1)$ . *(2 points)*