1) Random variable ξ has the following probability density function: $P(x)=\begin{cases} 2b, & 0 \le x < 1, \\ b, & 1 \le x \le 2, \\ 0, & x \notin [0,2]. \end{cases}$.

Calculate:

a) coefficient b, and distribution function $F_{\xi}(x)$; (1 *point*)

b) mathematical expectation $M\xi$, (1 *point*)

c) variance $D\xi$; (1 *point*)

d) probability that random variable ξ will become greater than ${}^{5}\!/_{6}$. (1 *point*)

2) Calculate $\int x \cdot e^{\frac{-x^2}{2}} \cdot dx$ (1 *point*)

3) Find the solution of the given matrix equation:

$$\begin{pmatrix} 1 & -1 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}.$$

(2 *points*)

4) Two players **A** and **B** roll the pair of ordinary dice in turn. Player **A** rolls first. To win, a player must sow a sum of dots on the two dice equal to ten. Game is going on until somebody wins. Compute the probability for the second player **B** to win. (2 *points*)

5) Find the equation of the tangent line and normal line to the given curve

$y = x \cdot \ln x$ at the point $x_0 = 1$. (2 *points*)

6) Compute the equations of asymptotes for the graph of the function:

$$y = \frac{x^2}{\sqrt{x^2+2}} \text{ (2 } \textit{points}\text{)} \qquad \text{(2 } \textit{points}\text{)}$$

7) For each of the following points $x = 2; y = 1$ and $x = -1; y = -2$ determine whether it is local max point, local min point or neither for the given function:

$$z = x^3 + 3xy^2 - 15x - 12y\,.$$ (4 *points*)

8) Mega Memory Devices, a firm that assembles memory boards for personal computers, buys 60% of its memory chips from supplier A and the remainder from supplier B. Supplier A produces memory chips that are 5% defective, and B produces 10% defective. A memory chip is selected at random from the inventory. A test of the chip shows that it is defective. What is the probability that the chip was supplied by A? *(2 points)*

9) For the given function $u = 3x^{1/3}y^{2/3}$ compute the gradient at point $M = (1;1)$ and the directional derivative of the function at $M\,(1;1)$ in the direction of the calculated gradient. (3 *points*)

10) Solve the given initial value problem:

$$\begin{cases} y' = \dfrac{y}{x} + \cos\dfrac{y}{x} \\ y(1) = 0 \end{cases}$$

(3 *points*)

11) Find the general solution of the following differential equations:

a) $y'' - y' - 2y = 0$ (1 *point*)

b) $y'' - y' - 2y = e^{x}$ (2 *point*)

c) $y'' - y' - 2y = 4e^{-x}$ (2 *points*)