

1) Find the equation of the tangent line and normal line to the given curve

$$y = x \cdot \ln x \text{ at the point } x_0 = 1.$$

(2 points)

2) Calculate  $\int x \cdot e^{\frac{-x^2}{2}} \cdot dx$

(1 point)

3) Random variable  $\xi$  has the following probability density function:  $P(x) = \begin{cases} 2a, & 0 \leq x < 1, \\ a, & 1 \leq x \leq 2, \\ 0, & x \notin [0, 2]. \end{cases}$

Calculate:

a) coefficient  $a$ , and distribution function  $F_{\xi}(x)$ ; (1 point)

b) mathematical expectation  $M\xi$ , (1 point)

c) variance  $D\xi$ ; (1 point)

d) probability that random variable  $\xi$  will become greater than  $\frac{5}{6}$ . (1 point)

4) Find the solution of the given matrix equation:

$$\begin{pmatrix} 1 & -1 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}.$$

(2 points)

5) Two players **A** and **B** roll the pair of ordinary dice in turn. Player **A** rolls first. To win, a player must show a sum of dots on the two dice equal to four. Game is going on until somebody wins. Compute the probability for the second player **B** to win. (2 points)

- 6) For the given function  $u = 3x^{2/3}y^{1/3}$  compute the gradient at point  $M = (1;1)$  and the directional derivative of the function at  $M (1;1)$  in the direction of the calculated gradient. (3 points)

- 7) Compute the equations of asymptotes for the graph of the function:

$$y = \frac{x^2}{\sqrt{x^2 + 2}}$$

(2 points)

- 8) For each of the following points  $x = -2; y = -1$  and  $x = 1; y = 2$  determine whether it is local max point, local min point or neither for the given function:

$$z = x^3 + 3xy^2 - 15x - 12y. \quad (4 \text{ points})$$

- 9) Mega Memory Devices, a firm that assembles memory boards for personal computers, buys 60% of its memory chips from supplier A and the remainder from supplier B. Supplier A produces memory chips that are 5% defective, and B produces 10% defective. A memory chip is selected at random from the inventory. A test of the chip shows that it is defective. What is the probability that the chip was supplied by B? *(2 points)*

10) Find the general solution of the following differential equations:

a)  $y'' - y' - 2y = 0$

(1 point)

b)  $y'' - y' - 2y = e^x$

(2 point)

c)  $y'' - y' - 2y = 4e^{-x}$

(2 points)

11) Solve the given initial value problem:

$$\begin{cases} y' = \frac{y}{x} + \cos \frac{y}{x} \\ y(1) = 0 \end{cases}$$

(3 points)