Kyiv School of Economics

July 2, 2012

Admission exam in Mathematics
(Both versions together)

1. A

   Sets A and B are given as: $A = \{x \in \mathbb{R} : x^2 + 2x - 3 \geq 0\}$; $B = \{x \in \mathbb{R} : 2 + x - x^2 > 0\}$.
   Find $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$.

   B

   Sets A and B are given as: $A = \{x \in \mathbb{R} : x^2 + x - 6 \geq 0\}$; $B = \{x \in \mathbb{R} : 3 + 2x - x^2 > 0\}$.
   Find $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$.

2. A

   Find the minimal distance between the line $y = 2x - 3$ and the point of origin $(0, 0)$.

   B

   Find the minimal distance between the line $y = 5 - \frac{1}{2}x$ and the point of origin $(0, 0)$.

3. A

   (a) Find such a non-zero matrix $\mathbf{X}$ so that the vector $\mathbf{a} = \mathbf{Xb} = \begin{pmatrix} m + 2n \\ n + 2k \\ k - 2m \end{pmatrix}$, if the vector $\mathbf{b} = \begin{pmatrix} m \\ n \\ k \end{pmatrix}$

   (b) Is matrix $\mathbf{X}$ invertible?

   B

   (a) Find such a non-zero matrix $\mathbf{A}$ so that the vector $\mathbf{b} = \mathbf{Ac} = \begin{pmatrix} x - y \\ y - z \\ z - x \end{pmatrix}$, if the vector $\mathbf{c} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

   (b) Is matrix $\mathbf{A}$ invertible?

4. A

   It is known that

   $$\int_n^{n+1} f(x)\,dx = n(n-1).$$

   Find

   $$\int_{-2}^{3} f(x)\,dx$$

   B

   It is known that

   $$\int_n^{n+1} f(x)\,dx = n^2 + n.$$

   Find

   $$\int_{-1}^{4} f(x)\,dx$$

5. A

   A random variable X has the following distribution function:

X	-1	-2	0	1
$P(X)$	0.2	0.1	k	0.4

   (a) Find k.

   (b) Find $\mu(X)$ (the *mean* of X).

   (c) Construct a probability distribution function for the function $Y(X) = X^2 - 1$

   (d) Find $P(Y > 0)$

   (e) Find the $\mu(Y)$. Is it true that $\mu(Y) = Y(\mu(X))$?

   B

   A random variable Y has the following distribution function:

Y	-1	0	2	3
$P(X)$	0.2	0.5	m	0.1

   (a) Find m.

   (b) Find $\mu(Y)$ (the *mean* of Y).

   (c) Construct a probability distribution function for the function $Z(Y) = Y^2 - 2Y$

   (d) Find $P(Z > 0)$

   (e) Find the $\mu(Z)$. Is it true that $\mu(Z) = Z(\mu(Y))$?

6. A

   Find the first and second partial derivatives of the following function:

$$z(x, y) = x^{\frac{1}{3}} y^{\frac{2}{3}}$$

   B Find the first and second partial derivatives of the following function:

$$z(x, y) = x^{\frac{3}{4}} y^{\frac{1}{4}}$$