Kyiv School of Economics

July 1, 2013

Admission exam in Mathematics
Version A

General instructions (read carefully!):

- You should NOT open the exam before your proctor says so.

- The exam has **15** problems and **8** pages. All problems will be weighted equally.

- You have **75** minutes for this exam.

- All problems are quite simple, but not necessarily straightforward. If you have spent more than 5 minutes per each problem (on average), it may suggest that you are moving in a wrong direction.

- The answer for each problem is a number or a short expression. Write down your answer in the Answer sheet.

- Provide explanations and the solution path right in the exam book. In the case of a wrong answer, a partial credit may be given based on your explanations.

- Please, write clearly. I will not be able to grade your work if I fail to read your writing.

- **Cheating on any exam automatically invalidates all your admission tests!**

- You can use a back of any page for your draft notes.

- GOOD LUCK!!!

Answer Sheet

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

1. The GDP (Gross Domestic Product) of a country has increased by 5% in Year 1, then increased by another 7% in Year 2, and then dropped by 10% in Year 3. Has the GDP increased, decreased or remained the same during the course of these three years? An intuitive explanation or an arithmetical expression will suffice for the answer.

2. Solve the system of linear equations:
$$2x - 3y \ = -7$$
$$6y - 4x \ = 14$$

3. Consider a new operator **mod**. The result of the **mod** operation $(a \bmod b)$ is the remainder of the division of a by b. Find the value of the expression

$$((5 \bmod 2) \bmod (7 \bmod 3))$$

4. An apple and a banana together cost $1.20. The apple costs $1.00 more than the banana. What is the price of the banana?

5. Let $f(x) = x^2 - 2x - 3$. For what values of x is it true that $f(x + 1) = f(x) + f(1)$?

6. Let $h(x) = \dfrac{1}{x} - x$. Find $h(h(1))$.

7. Compute the first order derivative of the function $y = \ln 2x + 3x^2 - 2$.

8. A firm produces $Q = 2 \ln L$ units of a product when L units of labor are employed. If the price of the product is 125 euros, and the price per unit of labor is 50 euros, what value of L maximizes profits?

9. Let $f(x) = \dfrac{5}{\sqrt{2x^2 + 5}}$. Find all values of x that maximize the function on the interval $[-3, 4]$.

10. Evaluate the integral:

$$\int_{-2}^{3} (3x^2 - 7x + 2)dx$$

11. Let $f(x, y) = \frac{1}{2}\ln(x^2 + y^2)$. Find $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$.

12. If matrix $\mathbf{A} = \begin{pmatrix} 1 & -5 \\ 0 & 1 \end{pmatrix}$, find $\mathbf{A}^4$.

13. For what values of x are vectors $\mathbf{a} = (x, -3x, 1)$ and $\mathbf{b} = (x, -x, 2x)$ *orthogonal*?

14. There are 5 male students and 7 female students in a room. If you randomly draw one student, what is the probability that she is a female?

15. Random variable w has the following probability density function:

$$p(w) = \begin{cases} 2cw, & \text{if } w \in [0, 3]; \\ 0, & \text{if } w \notin [0, 3]. \end{cases}$$

Determine coefficient c.