Kyiv School of Economics Kyiv School of Economics Admission Exam in Mathematics Version A

General instructions (read carefully!):

• You should NOT open the exam before your proctor says so.

• The exam has 15 problems and 8 pages. All problems will be weighted equally.

• You have 75 minutes for this exam.

• The answer to each problem is a number or a short expression. Write down your answers in the Answer sheet. However, please, provide, in the exam book, detailed explanations of how the answers have been attained.

• In the case of a wrong answer, a partial credit may be given based on your explanations.

• Please, write legibly (readably).

• Cheating on any exam automatically invalidates all your admission tests!

• You can use the back of any page for your draft notes.

YOUR NAME_____

Answer Sheet
1
2
3
4
5
6
7
8
9
10

1. Suppose a coin is flipped 5 times. What is the probability of getting at least 2 heads?

2. An urn contains 4 red marbles and 5 black marbles. Three marbles are drawn without replacement from the urn. What is the probability that all the marbles are black?

3. Evaluate the following integral

$$\int_{0}^{\infty} e^{-x} x^{n} dx \ (n \in \{0, 1, 2, 3, \ldots\})$$

4. Compute $\lim_{x\to 0} \frac{(x+1)\cos x - 1}{x}$

5. In a competitive market where the supply price (in hryvnias) is p = 3+0.25q and the demand price (in hryvnias) is p = 15-0.75q, the government imposes a per-unit tax of 4 hryvnias. What will be the tax revenue raised?

6. Evaluate the following integral: $\frac{4}{2}$

$$\int_{1}^{1} \frac{(\sqrt{x}+2)^3}{\sqrt{x}} dx$$

7. Solve the following equation $x^2 - 2|x| - 3 = 0$.

8. Find the intervals where the following function is increasing: $y = 4x - 5\ln(x^2 + 1)$.

9. A company uses inputs K and L to manufacture goods A and B. It has available 200 units of K and 180 units of L and the input requirements are:

10 units of K plus 30 units of L for each unit of A,

25 units of K plus 15 units of L for each unit of B.

If the per-unit profit is 80 hryvnias for A and 30 hryvnias for B, what combination of A and B should it produce to maximize profit?

10. Let x denote the temperature in degrees Centigrade and let y denote the temperature in degrees Fahrenheit. We know that x and y are linearly related, that 0° Centigrade or 32° Fahrenheit is the freezing temperature of water and that 100° Centigrade or 212° Fahrenheit is the boiling temperature of water. Find the equation (y = f(x)) which relates degrees Fahrenheit to degrees Centigrade.