A NEOCLASSICAL THEORY OF
WAGE ARREARS IN TRANSITION
ECONOMIES

by

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Abstract

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In the thesis I try to evaluate the losses of society from wage arrears phenomenon. For the purpose it was used the model developed by Lilia and Serguei Maliar. It is a standard one-sector neoclassical growth model. The neoclassical firms in transition make losses and use wage arrears as the survival strategy. At the agents' level, the randomness in the timing and extent of wage payments acts as idiosyncratic shocks to earnings. The model was calibrated for Ukrainian data and significant losses of consumption and social welfare were defined.

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GLOSSARY

Word. [Click and type definition here.]

Chapter 1

Introduction

Transition countries produced a huge number of economic problems that were interesting for economists to investigate. The problem that attracted my attention is wage arrears phenomenon. Arrears were widely investigated by many economists e.g. Alfandari (1996), Lehman (1999), Desai and Idson (2000), Earle and Sabirianova (2000) and others. The large interest in this phenomenon can be easily explained – the arrears in such magnitude were not observed anywhere in the world before. From my point of view, wage arrears deserve to be called phenomenon; the reason is that the process when workers produce some product without receiving any reward for their efforts for a long period of time could not be considered as usual.

The following two questions appear to be of particular interest: what initiated wage arrears and supported them for a long period of time; what consequences the phenomenon had on the economies of the countries where it took place.

Many authors investigated the first point about the reasons of late payments. The existence of wage arrears was attributed to liquidity problems, such as the lack of external finance available to enterprises, and the non-payment by customers for the goods delivered (Alfandari and Shaffer, 1996, Clarke, 1998); to the opportunistic behavior of managers who practice wage payment delays to pursue their personnel interests, for instance, to make workers sell their shares of enterprises (Earle and Sabirianova, 2002); to the willingness of workers to accept wage-cuts in order to preserve their jobs (Layard and Richter, 1995); to the attempt of managers to extract tax concessions from the government (Alfandari and Shaffer, 1996); to the "survival" strategies of loss-making enterprises (Desai and Idson, 2000), etc.

Despite the fact that considerable attention was paid to the reasons of wage arrears and sufficient theory framework was provided, researchers are still not sure about the nature of the phenomenon.

Regarding the second point about the consequences of wage arrears, little interest was paid to this side of the problem. Only Lehmann and Wadsworth in their paper considered the influence of wage arrears on the distribution of earnings in Russia. However, the magnitude of the losses of society (even approximate) from the persistence of wage arrears is important. The reasoning of importance is that having information about magnitude of losses from different economic problems gives an opportunity to make a clear ordering of problems according to their loss-making power.

Even without calculations, I can state that wage arrears caused a significant negative influence on the level of worker's welfare in Ukraine. One can imagine himself (herself) waiting for wage payments to buy some present for his (her) child but instead of money you receive a promise of "100% sure" payment in the next month. In addition, the situation repeats, almost every month. Regarding the numbers, they are the following: during 1996-2001 the level of wage arrears was around 22% of quarterly output; in 1996 wage arrears constituted around 40% of the yearly salary (in other words, the average wage debt was equal to five monthly salaries); and they affected more than 60% of the labor force.

In my work I evaluate the role of arrears in the economic performance. For evaluation I use the model of wage arrears developed by Serguei and Lilia Maliar (2003) on the basis of the incomplete markets model of Aiyagari (1994). For the purpose two identical artificial economies are considered. In the first economy the wages are always paid and there is no uncertainty. All agents are equal, and all the variables are in the steady state. In the second economy, there are wage arrears. The wages of the agents are paid stochastically what leads to differences in consumption-saving behavior across agents. Then for stationary equilibrium the distributions of the individual quantities are computed and the corresponding aggregate quantities are found. Next the implications of the two economies are compared and thus it is found what is the role of arrears both at micro level (for individual consumption saving behavior) and macro level (for the aggregates and the social welfare).

For explanation of the determinants of the wage arrears phenomenon and for assessment of the effect of wage arrears on the individual consumption-savings behavior Lilia and Serguei Maliar developed a model and methodology for calculation. In the model wage arrears arise because firms, hit by negative shocks associated with transition, incur losses and cover the losses by underpaying wages to workers. It was assumed that wage arrears depreciate over time, which allows capture the fact that wage arrears in transition economies were often not indexed by inflation. At the individual level, the randomness in the timing and extent of wage payments act as idiosyncratic shocks to earnings. Markets are incomplete: agents cannot borrow beyond a certain limit and cannot insure themselves against idiosyncratic uncertainty. Thus, the consumer side of Maliars' economy is the same as the standard one-sector neoclassical growth model except that in Maliars' case, fluctuations in individual wages come from wage arrears shocks, while in the standard case, they come from productivity shocks.

The structure of the work is the following: in the ***literature review*** I suggest short overview of the most important researches on the topic of wage arrears; in ***the description of the data*** section it is provided the information about the source of the data used in the calculations and description of data processing; in ***the description of the model*** section Maliar's model is suggested; in the section of ***qualitative analysis*** the methodology of the model calibration is provided and the results of calibration are described; ***conclusion*** summarizes the main results; in ***appendix A*** the methodology of computation of the non-stochastic steady is described; in ***appendices B and C*** our current work on the improvement of the calibration procedures is described.

Chapter 2

Literature review

The problem of wage arrears is widely discussed in the economic literature. The main emphasis has been made on the question why payment delays appeared in such scales'. The majority of articles focus on Russia. However, there are many other Post Soviet Countries experiencing this problem.

Wage arrears are considered as a phenomenon because in developed economies payment delays are practiced only in two cases: in small start-up companies facing severe liquidity constraints and when an enterprise is going to be bankrupt. In other cases employees always receive their wages. In the case of Russia and Ukraine workers have not been receiving their wage for more than half a year and they still do not quit their jobs.

Lehmann, Wadsworth, and Acqusity (1999) regarded wage arrears as a pervasive feature of Russian economic life since 1994 affecting large scale of the workforce (between 40 and 70 %).

Several explanations for the existence of arrears have been proposed.

First, payment delays have been found to depend on the local labor market conditions. The logic is that in many regions workers have little opportunity to change workplace when experiencing payment delays. Monopsonic power of some enterprises is very large and transaction costs for workers are not zero so that they are not free to move to other region. Lehmann, Wadsworth, and Acquisiti (2000) state that the probability of payment delays at Moscow and St. Petersburg, where a worker has much more possibility to change job, is much less than at other regions of Russia.

Earle and Sabirianova (2002) state that previous payment delays at the region increase probability for firms at the same region to experience wage arrears at the next period.

Local labor market conditions become important because of low labor mobility. Of course, there are regions where the level of wage arrears is lower then on average but due to such factors as transaction costs, tenure, and fringe benefits workers do not migrate actively. The influence of wage arrears on labor mobility, however, is ambiguous: on the one hand, payment delays induce employees to quit their workplaces and look for another job but on the other hand, transaction costs, tenure, fringe benefits, and fear to loose the owed money make the workers to stay at their jobs (Earle and Sabirianova (2002)). Lehmann, Wadsworth, and Acquisiti (2000) state that the push effect of payment delays (impulse to quit) is not quite offset by pull effect (the inducement to stay and retain employment). So wage arrears are an incentive to quit but not very strong one.

Second, the behavior of managers is another reason of wage arrears. Under conditions of high inflation it is extremely profitable to delay payments and make the free money work somewhere else[1]. Moreover, Earle and Sabirianova (2002) argue that at the enterprises with workers-shareholders managers practice payment delays to make the workers sell their shares. The conclusion was made on the basis of positive relationship between employees – small – shareholders (less than 5%) and the probability of wage arrears incidence. Lehmann, Wadsworth, and Acquisiti (2000) indicate that ownership reduces the chance of wage arrears but a small share (less than 5%) in the firm raises the likelihood of arrears, other things equal. However,

[1] Short-term treasury bills were offered at extremely high interest rates – in 1995-96 declared interest rate on treasury bills was about 90 –140 % (pure yield on short-term treasury bills was about 40 – 60 %); even without any financial schemes "a loan with a zero interest rate" under two-digit inflation is extremely profitable

Lehmann did not make any conclusion about managers' behavior from this finding.

A lot of attention has been paid by researches[2] to the liquidity problems of enterprises. Financial problems are considered to be responsible for wage arrears. According to Clarke (1998): "The worst payment delays of wages is not found in enterprises which are bankrupt, but in the most prosperous and profitable enterprises in Russia. They do not pay wages not because they cannot afford to pay wages, but because they do not have the free money to pay wages". Alfandari and Schaffer (1996) found that there was no correlation between financial distress and wage arrears. However, they distinguish liquidity problems from financial distress and conclude that late payments are used for mitigating negative influence of liquidity difficulties. Earle and Sabirianova (2002) reported that there is negative influence of liquidity indexes[3] on wage arrears incidence. In other words, an enterprise with better liquidity indexes is less likely to practice payment delays.

The soft budget constraints that were widely used at the Soviet Union also stimulated payment delays. According to Earle and Sabirianova (2002) "old" firms (those that were established before the beginning of *perestroika* at the late 1980's) were much more likely to have arrears than those founded subsequently, although the problem was significant even among the latter, sometimes called *de novo* firms. The "new generation" of enterprises used to work without "soft budget constraints".

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2 Desai and Idson (2000), Lehmann, Wadsworth, and Acqusiti (1999), Earle and Sabirianova (2002).

3 The current ratio (current asset/current liabilities), liquidity ratio ([current assets - stock]/current liabilities), and export-output ratio. The current ratio and liquidity ratio are standard balance sheet indicators for firm liquidity. Current assets include stocks, accounts receivable, short-term financial investments, cash and other current assets. Stocks include raw materials, low-value and short-term assets, work in progress, and finished product in inventory. Current liabilities include loans, accounts payable, and other current liabilities.

One of the important sources of payment delays is budget deficit.[4] Teachers, bureaucrats, healthcare providers suffered wage arrears almost permanently. However, payment delays due to budget deficit took only about 15 % in the total amount of Ukrainian wage arrears. So budget deficit could not be defined as the major determinant of the phenomenon.

Researchers were also interested in investigation of the discrimination across individuals in the same enterprise. Although, firm characteristics dominate individual characteristics (Lehmann, Wadsworth, and Acquisiti (2000)), such factors as gender, job tenure, occupation, and age were defined as important in wage arrears process by many researchers.[5] Gerry (2001) when considering gender wage gap and wage arrears in Russia concluded the following: "…wage arrears – occurring with greater propensity and persistence amongst men than women – were a mechanism enabling women to be compensated for their loss from high wage discrimination ". According to Lehmann, Wadsworth, and Acquisiti (2000) workers with longer tenures have higher probabilities of experiencing wage arrears.

Wage arrears are not distributed equally among industries. Some sectors of economy were severely hit with payment delays. According to Lehmann, Wadsworth, and Acquisiti (2000) the highest part of workers experiencing wage arrears was at agriculture (80.6 %) then manufacturing, construction, mining, transport, and health/education (50-70 %).[6]

Also researches were interested in the consequences of wage arrears. For instance, the impacts of arrears on workers' labor supply decisions. Earle and Sabirianova (2002) reported that wage arrears do not influence transition to unemployment or out of the labor force quits because of high arrears in unemployment benefits and pensions. While Desai and Idson (1998) found

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[4] Lehmann, Wadsworth, and Acqusiti (1999), Earle and Sabirianova (2000).

[5] Desai and Idson (2000), Lehmann, Wadsworth, and Acqusiti (2000), Earle and Sabirianova (2002).

[6] The results are for Russian economy; however, the situation in Ukrainian economy is pretty much similar.

that the people who were owed wages were more likely to hold second job and were more likely to engage in supplemental individual economic activity. Thus Desai and Idson concluded that wage payment delays had distinct labor supply effects.

One important consequence of arrears is the reduction in disposable income and the incidence of family poverty. According to Desai and Idson (1998) payment delays not only reduce family disposable income but also cut into family wealth because wage arrears are not indexed. This means that even if workers were to be eventually repaid, they would have suffered a significant loss given the high monthly inflation. Desai and Idson (1998) reported that arrears increased the probability of incidence of poverty.

It is logical to consider the effect of arrears on savings. Desai and Idson (1998) found that "although people experiencing wage arrears were less likely to save (conditional on positive savings), people who were owed wages had average savings levels similar to those for people who were fully paid, and people did not seem to compensate for wage arrears by living off their savings". In other words, people were saving some constant part of their disposable income and were trying not to dissave.[7]

Alfandari and Schaffer (1996) argue that wage arrears are used by management in some firms to extract tax concession from the government. Tax evasion problem was also considered by Slinko (1999). He defined that late payments make people look for second job but due to high income-taxation they spend some time on their first workplace to pay taxes from the wage that is delayed. Thus workers also use wage arrears to "reduce their tax burden".

Existence of wage arrears for a long period of time stimulated researchers to find some equilibrium for wage arrears. Earle and Sabirianova

[7] Intra-family transfers, the sale of family assets in response to wage nonpayment, home production of food items partially mitigated the effect of wage arrears on the incidence of poverty (Desai and Idson (1998)).

(2000) presented the model of managerial choice of wage delays that implies a possibility of multiple equilibria in the level of arrears. They study three equilibria, distinguishing two that are stable – "punctual payment equilibrium" and the "late payment equilibrium" – and one unstable "critical mass equilibrium", a threshold of arrears in the local labor market beyond which even profitable firms may adopt in practice.

Obviously, wage arrears will affect the level of welfare of the workers. But the question of 'to what extent' payment delays will affect welfare attracted attention of researchers only recently.[8] When investigating impact of wage arrears on the distribution of earnings, Lehmann and Wadsworth (2001) constructed a counterfactual wage distribution without wage arrears and compared it to a distribution of earnings in case of arrears. The conclusion from the calculations was the following: "While wage arrears were not responsible for the large scale increase in inequality, the estimates in our paper suggest that they may have been partly responsible for the failure of inequality to fall back following the unanticipated price shocks in the first half of the nineties". In other words, wage arrears were not the main determinant of large inequality in the earning distribution.

Results of researches on reasons of wage arrears provided a powerful base for investigating the consequences of wage arrears. In the work, we calculate losses of society from payment delays for ukrainian economy. In the next section, it is described the data used in the estimation process.

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[8] Hartmut, Lehmann, Jonathan, Wadsworth, 'Wage arrears and the distribution of earnings in Russia', IZA Discussion Paper # 410, December 2001.

Chapter 3

Description of the data

The household survey "Ukraine 96" was carried out by Kiev International Institute of Sociology, and it contains information on 5403 Ukrainian households. From the whole sample it was selected a subsample where household heads were employed. Attention was restricted to wages and wage arrears of the household heads on the main job (the one that brings the largest income). The agent's monthly wage was defined as the sum of both monetary and non-monetary compensation, as estimated by respondents, specifically,

ω^{month} = *the net working compensation* + *the estimated cost of goods received* + *the estimated cost of privileges*

The distribution of yearly wages was constructed by bootstrapping[9]. Specifically, the yearly wage was computed as a sum of 12 random draws from the constructed distribution of monthly wages, i.e., $\omega^{year} = \sum_{i=1}^{12} \omega_i$, where $\omega_i \sim \{\omega^{month}\}$. The mean and standard deviation of wages reported in *Table 1* are computed over the sample with 1000000 observations.

The mean and standard deviation of wage arrears in *Table 1* are those of the amount of money that the owner or the administration owes to the household head on the main job.

To draw the empirical distribution of wealth and wage arrears in *Figure 2,* we used the individuals' subjective evaluations of their material status

[9] . In the survey, we have the information about the money received during the last 30 days. There are individuals that did not receive any payments during that period. So if multiplying monthly wages (we consider the payments as approximation of wages) on 12 we will have individuals with no wages for the whole year.

relative to the one of other people in their city (village, town). This variable takes values from 1 to 7, which correspond to evaluations "much lower than average", "lower than average", "a bit lower than average", "average", "a bit higher than average", "higher than average" and "much higher than average", respectively.

To construct the educational and profession-qualification groups, employed for the sensitivity experiments, we used the education and profession (qualification) of the household head.

The Ukrainian aggregate data such as the GNP, personal consumption, wages government expenditures, wage arrears, gross payables, gross receivables, population, CPI, and PPI are quarterly time series coming from UEPLAC (2001). All the series except of the one for wage arrears range from 1994:1 to 2001:4; the series for wage arrears range from 1996:3 to 2001:4. The nominal series were converted into real one by using the CPI.

As a measure of output in the model, Y, the GNP was used. The aggregate consumption in the model, *C*, was defined as the sum of the personal and government consumption. The time series data on the government consumption is not available. However, UEPLAC (2001) provides a detailed budget of the Ukrainian government for the year 1998, which allows to roughly estimate how the total government expenditures are subdivided between consumption and investment. The government investments were defined as the sum of the government expenditures on the R&D, education, construction, health care, telecommunication, transportation and the reserve funds. The rest of the government spending was defined as the government consumption. With this definition, we have that about half of the government expenditures goes to investment and the other goes to consumption (the exact estimates of the investment and consumption shares in the total government expenditures are equal to 0.48 and 0.52, respectively). Therefore, the series for consumption are constructed by summing up personal consumption and 0.52 of the total government expenditures. The aggregate labor income in the model, Y^L, is defined as the aggregate wage bill excluding the wages paid by collective agricultural enterprises. The estimates of the Ukrainian capital stock, *K*, are available for the years 1996, 1998, 1999 and 2000 from Derzhcomstat (2000). This variable is converted into the real terms by using the PPI.

Chapter 4

The description of the model

The model was developed by Lilia and Serguei Maliar(2003) on the basis of the incomplete markets model of Aiyagari (1994).

Time is discrete and the horizon is infinite, $t \in T$, where $T = \{0,1,2...\}$. The economy consists of a continuum of firms and a continuum of infinitely lived consumers. Both, firms and consumers, have their names uniformly distributed on a closed interval [0,1].[10]

4.1 The firms

Firms own identical production technologies that convert capital and labor into output. Each firm chooses demand for capital input, K_t, and labor input, N_t, to maximize period-by-period profits, taking the interest rate, R, and wage, W, as given.[11] The level of output depends on additive idiosyncratic shock, θ_t, which is independently and identically distributed across firms and over time. Thus, the firm solves the following maximization problem:

$$\max_{K_t, N_t} \; E\left[F(K_t, N_t) - dK_t - RK_t - WN_t - \theta_t\right] \quad , \quad (1)$$

where E is the unconditional expectation, and $d \in (0,1]$ is the depreciation rate of capital. The production function F has constant returns

[10] This assumption implies that the average and aggregate quantities coincide.

[11] Here and further in the text, variables whose equilibrium values are common for all firms are denoted by capital letters, and those whose values are firm-specific are denoted by small letters. The same type of notation will be used for consumers.

to scale, is strictly increasing, strictly concave, continuously differentiable and satisfies the appropriate Inada conditions.

The profit maximization conditions, which follow from (1), imply that the factor prices are equal to the corresponding marginal products:

$$R = F_1(K_t, N_t) - d, \qquad W = F_2(K_t, N_t), \qquad (2)$$

where F_1 and F_2 are the first-order partial derivatives of the production function f with respect to capital and labor inputs, respectively.

With the assumption of constant returns to scale and in the absence of idiosyncratic shocks, $\theta_t = 0$, the firm makes zero profit. When $\theta_t > 0$ $(\theta_t < 0)$, the firm makes negative (positive) profit. We assume that the firm is always sufficient to cover depreciation of capital and its rental price, i.e., $F(K_t, N_t) - \theta_t \geq (d + R)K_t$. Thus, the effective wage per unit of labor is

$$\omega_t = [F(K_t, N_t) - (d + R)K_t] / N_t = W - \theta_t / N_t \qquad (3)$$

Wages that are (over-paid) under-paid to workers are (subtracted from) added to the stock of wage arrears,

$$q_t = q_{t-1}(1 - d_q) + \theta_t, \qquad (4)$$

where q_t denotes the stock of wage arrears at the end of period t. We assume that q_t is bounded, i.e., $q_t \in \vartheta \equiv [0, \bar{q}] \subset \Re$ for all t. The depreciation rate of wage arrears is $d_q \in (0,1]$.

The assumption that the stock of wage arrears depreciates, allows us to account for the fact that, in transaction economies, wage arrears were not appropriately indexed by inflation, so that a delay in wage payment resulted in the reduction of the real value of wage debt. Furthermore, some wage arrears were not paid at all, either because firms go bankrupt or because workers quit their jobs and loose the right to claim the debt.

In order to capture the fact that a typical firm in a transition economy is a loss-maker, we shall assume that the idiosyncratic shock, θ_t, is, on average, positive, $\mathrm{E}[\theta_t] > 0$. By assumption, the process for θ_t is exogenous and cannot be affected by actions of the firm. However, note that even if we allow the firm to reduce θ_t at zero cost, it would have no incentives to do so. This is because the firm shifts all losses to workers and makes the same effective (zero) profit, independently of the amount of wage arrears.[12]

4.2 The consumers

Each agent is matched with one firm and supplies inelastically one unit of her (non-valued) time to production, N_t=1. The contractual wage of agent is *W,* however, her effective wage is ω_t. The evolution of the debt of the firm to the worker (wage arrears) is described by (4). Our assumptions imply that the distribution of effective wages is the same for all firms, and thus, agents have no incentives to change their jobs even if they are paid less than the amount specified in the contract.

The agent saves in the form of real assets. Income from assets is Ra_t, where a_t is the current individual asset holding. Assets are restricted to being in the set $A = [-\underline{a}, \bar{a}] \subset \Re$, i.e., the agent is allowed to borrow only up to a certain amount $\underline{a} \geq 0$. The agent seeks to maximize the expected discounted sum of one-period utilities by choosing an optimal path for consumption,

[12] In fact, there is ample evidence in the empirical literature that managers tend to exploit the phenomenon of wage arrears in their own interests, e.g., to extract tax concessions from the government (Alfabdari and Shaffer, 1996), to make the workers sell their shares of the firm (Earle and Sabirianova, 2002).

$\{c_t\}_{t=0}^{\infty}$. The period utility function *u(c)* is continuously differentiable, strictly increasing, strictly concave and satisfies Inada type of condition $\lim_{c \to 0} u'(c) = \infty$ (consumption is restricted to be non-negative). Consequently, the problem of the agent is as follows:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t u(c_t) \tag{5}$$

subject to

$$c_t + a_{t+1} = \omega_t + (1+R)a_{t+1}, \tag{6}$$

$$a_{t+1} \geq \underline{a}, \tag{7}$$

where initial condition (a_0, q_{t-1}, ω_0) is given. Here, E_t denotes the expectation, conditional on all information about the agent's wage payments available at *t*, and $\delta \in (0,1)$ is the discount factor. To ensure the existence of a solution to the consumer's problem, we shall that $\delta(1+R) < 1$.

The consumer's problem, in the model with wage arrears, is identical to the one in the standard one-sector neoclassical growth model by Aiyagari (1994). The exception is that in Maliar's model fluctuations in the individual wage is the result of delays in wage payments and not of idiosyncratic shocks to productivity, as assumed in Aiyagari (1994). Also it is necessary to note that that in Maliar's economy the individual state is characterized by three state variables, (a_t, q_{t-1}, ω_t), while in Aiyagari (1994) economy, it is described by two state variables, (a_t, s_t), where s_t is an idiosyncratic shock to productivity.

Now the attention is restricted to a recursive solution of the agent's problem, such that the agent makes consumption-savings decisions according to the same decision rule in all periods. It turns out that the number of state variables can be reduced from three to two with the following change of variables:

$$\hat{a}_t = a_t + \underline{\underline{a}}, \qquad (8)$$

$$z_t = \omega_t + (1+R)\hat{a}_t - R\underline{\underline{a}}. \qquad (9)$$

The variable z_t can be interpreted as the total amount of resources that is available to the agent in period *t*. In terms of (8) and (9), constraints (6) and (7) could be re-written as follows:

$$c_t + \hat{a}_{t+1} = z_t, \qquad (10)$$

$$\hat{a}_t \geq 0. \qquad (11)$$

Let us denote by $V(z,q)$ the optimal value function for the agent with the total resources *z* and wage arrears *q*. The recursive formulation of the problem (3) – (7) is then as follows:

$$V(z,q) = \max_{\hat{a}' \in [\underline{\underline{a}}, \bar{\bar{a}}]} \left\{ u\left(z - \hat{a}' \right) + \delta E[V(z', q') \mid q] \right\} \qquad (12)$$

subject to

$$z' = \omega' + (1+R)\hat{a}' - R\underline{\underline{a}}. \qquad (13)$$

The problem (12), (13) defines the optimal asset demand function, $\hat{a}' \equiv A\ (z,\ q)$. The function is assumed to be unique, and also that it is continuous and differentiable.

After calculation of the Khun-Tucker conditions of the problem (12), (13), the Euler equation was obtained

$$u'(c) \geq \delta E\{u'(c')(1+R)\}, \qquad (14)$$

where $c = z - \hat{a}'$, and *u'* denotes the marginal utility of consumption. The Euler equation (14) holds with equality if the borrowing restriction is nonbinding, $a' > \underline{\underline{a}}$, and it holds with inequality if the limit on borrowing is reached, $a' = \underline{\underline{a}}$.

4.3 Equilibrium

Let λ be a probability measure defined on *B*, where *B* denotes the Borel subset of the set of all possible individual states $Z \times Q$. For all B∈ *B*, $\lambda_t(B)$ is the mass of agents whose individual states lie in *B* at time *t*. Given that λ_t is a probability measure, the total mass of agents is equal to 1.

Denote by $P(z,q,B)$ the conditional probability that an agent with state *(z, q)* will have an individual state lying in set *B* in the next period. The function *P* is defined as

$$P(z,q,B) = \Pr ob(\{q' \in Q : [z', q'] \in B\} \mid q).$$

Then, the law of motion of λ_t is $\lambda_{t+1}(B) = \int_{Z \times Q} P(z,q,B) d\lambda_t$ for all $t \in T$ and all B∈ *B*.

The fact that there is a continuum of agents guarantees that the mass of agents with the shock *q'* at *t+1* and the shock *q* at *t* is equal to the conditional probability, $\Pr ob(q' \mid q)$. Since q_{t+1} follows a first-order Markov process, such probability depends only on the recent past and is the same in all periods. Hence, the aggregate amount of wage arrears is constant.

Maliar's model only studies the equilibria in which the period-*t+1* probability measure λ_{t+1} is the same as the period-*t* probability measure λ_t, for all $t \in T$. In this case, the probability measure is stationary and is denoted by λ^*.

The stationarity of λ^* implies that the aggregate capital stock is constant, $K = \int_{Z\times Q} a_t d\lambda^*$ for all *t*, (even though the asset holdings of each agent vary stochastically over time).

Definition: A stationary equilibrium is defined as a stationary probability measure λ^*, an optional asset function *A (z, q)*, and positive real numbers *(K, Q, R, W)* such that:

(1) λ^* satisfies $\lambda^* = \int_{Z\times Q} P(z, q, B) d\lambda^*$ for all B ∈ *B*;

(2) A (z,q) solves (13), (14) for a given pair of prices *(R,W)*;

(3) (R, W) satisfies the profit maximization conditions in (2) with $N_t = 1$;

(4) *Q* is the average of the agents' wage arrears: $Q = \int_{Q} q_t d\lambda^*$;

(5) *K* is the average of the agents' asset holdings: $K = \int_{Z\times Q} a_t d\lambda^*$.

Quatitative analysis

In the section it described the methology of the numerical study and the results are presented. The description of the Ukrainian aggregate and household data is provided in *Chapter3*.

5.1 Methodology

For the model's period we chose one year. To reproduce several basic observations on the Ukrainian economy I calibrated the model. With the assumption of stationarity of the wage and wage arrears distributions, it is easy to show that[13]

$$d_q = 1 - \sqrt{1 - \sigma_\omega^2 / \sigma_q^2} \tag{15}$$

$$W = \Omega + Q d_q \tag{16}$$

where Ω and σ_ω (Q and σ_q) are the mean and the standard deviation of the wage distribution (the wage arrears distribution), respectively.[14] Thus, the ratio of the average effective wage to the contractual wage is given by

$$\psi_q \equiv \frac{\Omega}{W} = \frac{\Omega}{\Omega + Q d_q}. \tag{17}$$

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[13] $q_t = q_{t-1}(1 - d_q) + \theta_t = q_{t-1}(1 - d_q) + W - \omega_t$;
$\mathrm{var}(q_t) = \mathrm{var}(q_{t-1})(1 - d_q)^2 + \mathrm{var}(\omega_t) - 2(1 - d_q)\mathrm{cov}(q_t, \omega_t)$;
$\mathrm{cov}(q_t, \omega_t) = 0$; $\mathrm{var}(q_t) = \mathrm{var}(q_{t-1})$;
$d_q = 1 - \sqrt{1 - \frac{\mathrm{var}(\omega_t)}{\mathrm{var}(q_t)}}$

[14] To derive (15) it was assumed that the current individual wages are uncorrelated with past wage arrears. To check the empirical validity of this assumption, it was computed this correlation with Ukrainian household data and found that it is, indeed, low (about 0.05).

Ω, Q, σ_ω, and σ_q were calculated by using the data from the "Ukraine -96" household survey and d_q, W and ψ_q were computed according to (15), (16) and (17), respectively. The results are reported in the upper row of *Table 1*.

The production function was assumed to be of the Cobb-Douglas type, $F(K,N)=K^{\alpha}N^{1-\alpha}=F(K,1)=K^{\alpha}$. The remaining parameters were calibrated $\{\alpha,d,\delta\}$ so that, in the non-stochastic steady state, the model reproduces the following four statistics of the Ukrainian economy: the share of labor income in production, Y^L/Y, the consumption to output ratio, C/Y, the capital to output ratio, K/Y, and the wage arrears to output ratio, Q/Y. In the absence of wage arrears, it would be that the share of labor in the production function is $1-\alpha=(Y^L/Y)$ where Y^L and Y are the non-stochastic steady state income of labor and output, respectively. With wage arrears, agents do not get all the labor income earned but only a fraction of it, ψ_q, so that

$$\alpha=1-\frac{1}{\psi_q}\left(Y^L/Y\right). \tag{18}$$

Furthermore, equations (2) - (4), (6) yield

$$d=\frac{1-(C/Y)-d_q(Q/Y)}{(K/Y)}. \tag{19}$$

where C/Y, K/Y, and Q/Y are non-stochastic steady state ratios of consumption to output, capital to output, and arrears to output, respectively.

Finally, the Euler equation (14) implies

$$\delta = \frac{1}{1 - d + \alpha(Y/K)}. \tag{20}$$

The ratios Y^L/Y, C/Y, K/Y and Q/Y were estimated from the Ukrainian time series data and α, d, and δ were computed from equations (18), (19) and (20), respectively. The results are reported in the lower row of *Table 1*.

The debit limit is set at zero, $\underline{a} = 0$. The agent's momentary utility function is assumed to be of the Constant Relative Risk-Aversion (CRRA) type, $u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$ with $\gamma > 0$. The coefficient of relative risk-aversion, γ, is not identified by the calibration procedure. Four alternative values of these parameters were considered such as $\gamma \in \{0.5, 1, 3, 10\}$. The benchmark value is $\gamma = 1$, which corresponds to the limiting logarithmic case, $u(c) = \ln(c)$.

To compute numerical solutions, it was assumed that individual effective wages are drawn from a Normal distribution, $\omega_t \sim N(\Omega, \sigma_\omega^2)$ and the process for wage arrears (4) was approximated by a 5-state Markov chain. The resulting approximation is illustrated in *Figures* 3 and 4. In addition, it is provided the matrix of the transitional probabilities and the corresponding unconditional probabilities of states in *Table* 2. A description of solution procedure is elaborated in Appendix B. The properties of the solutions obtained in *Figures* 5-8 and the statistics generated by the model is reported in *Table* 3.

5.2 Results

The quantitative results were obtained from the model solved by Lilia and Serguei Maliar. They also developed a MATLAB program that produced the results suggested below. First of all, there was defined a benchmark model for calculations (see column "BM" in *Table 3*). The distributional prediction of the model could be illustrated by several inequality measures of the distributions of wages, assets and consumption. It could be seen that due to idiosyncratic uncertainty a significant dispersion in effective wages across workers takes place. For instance, the bottom 40% wage group gets 20% of the total wages in the economy, while the top 1% wage group earns 2% of the total wages; the Gini coefficient of the wage distribution is 0.27. As a result of wage inequality we can observe high inequality in assets and consumption. The dispersion in assets across agents is much higher than that in consumption. For example, the bottom 40% asset group holds 21.8% of the total assets, and the top 1% asset group holds 2.6%, while the respective consumption groups consume 33% and 1.6% of the total amount of consumption. The same tendency could be observed for the normalized standard deviations and the Gini coefficients. For the asset distribution the normalized standard deviation is 0.496 and Gini coefficient – 0.275, while for the consumption distribution – 0.165 and 0.105, respectively. [15] Hence, the model predicts that risk-averse agents smooth consumption fluctuations by accumulating assets in high-wage states and dissaving in low-wage states.[16]

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15 The dergees of wealth, income and consumption inequality in Maliar's model with wage arrears are similar to those generated by the standard neoclassical growth model with productivity shocks (see, e.g., Aiyagari, 1994)

16 This implication of the model agrees with the empirical findings of desai and Idson (2000) that in Russia, agents who were owed wages were more likely to engage "in various forms of dissaving including borrowing, selling family consumer durable items and other assets, and drawing down accumulated savings" (Desai and Idson, 2000, p. 218).

Figures 7 and 8 shows the simulated probability distributions of assets and wage arrears, and consumption and wage arrears, respectively. The *Figures* 7 and 8 support the idea that assets are more dispersed across agents than consumption. Consumption is always positive while assets occasionally reach a zero borrowing limit. Despite the fact that the available household survey does not contain information about the individual consumption and asset holdings, it was possible to approximate the level of asset holdings from the respondents' own evaluation of their material status (see the description of the data). In *Figure* 2 the joint distribution of assets and wage arrears is depicted (it is considered to be the empirical distribution). From *Figures* 2 and 7 one can see that there is a certain degree of similarity between the empirical and simulated distributions.

The next step is to explore how wage arrears affect the aggregate model's variables. Let us consider the average income, which is given by the sum of capital and labor income,

$$I = \int_{Z\times Q} (Ra_t + \omega_t) d\lambda^* = RK + \Omega . \tag{21}$$

Concerning the capital income, RK, uncertainty about the time and amount of wage payments, together with the restriction on borrowing, makes the agents increase their savings. As it can be seen from *Table 3* , the rise in the aggregate capital stock due to precautionary savings, ΔK , amounts to 2.936%.[17] The interest rate is inversely related to the capital stock, $R = \alpha K^{\alpha-1} - d$, so that its value is lower in the economy with wage arrears

[17] Lilia and Serguei Maliar denoted by ΔX the percentage differance between the mean of a variable X in the model with wage arrears and its non-stochastic steady state value, X^{SS} , i.e., $\Delta X \equiv \frac{X - X^{SS}}{X^{SS}} \times 100$. The stochastic steady state can be computed analytically. For instance, the steady state capital stock can be computed from the steady state exression of the Euler equation, $1 = \delta\left(1 - d + \alpha\left(K^{SS}\right)^{\alpha-1}\right)$, the other aggregate variables can also be computed.

than in the one without wage arrears. The total effect of wage arrears on capital income is negative, $\Delta(RK) = -0.985\%$.

Labor income is given by (16) with $W = (1-\alpha)K^{1-\alpha}$. There are two effects here. Firstly, the marginal product of labor goes up because capital stock rises due to precautionary savings, $\Delta W = 1.387\%$. Secondly, the wage reduces by the amount of $d_q Q$ because of the depreciation of wage arrears. The second effect dominates the first one, $\Delta\Omega = -20.360\%$.

It turns out that the above decrease in both capital and labor income leads to a significant reduction in total income, $\Delta I = -15.120\%$. As follows from the previous discussion, the main determinant of the income loss in the model is a very large depreciation rate of wage arrears, $d_q = 0.474$. Indeed, the estimates imply that about a half of the wage bill, which was not paid to the agent in time, is lost after a one-year delay. The high depreciation rate was obtained from wage arrears is presumably explained by a high inflation rate in Ukraine, which was, on average, equal to 63,40 % per year over the period 1994-2001.[18]

The next step is to evaluate the social welfare loss resulting from wage arrears. There are two possible reasons of the fall of social welfare. Firstly, the reduction in aggregate income pushes the aggregate consumption down. Secondly, because of idiosyncratic uncertainty about wage payments, individual consumption is volatile, which is disliked by risk-averse agents. In the benchmark case, the aggregate consumption loss is $\Delta C = -12.845\%$. To measure the cost of consumption fluctuations, the following procedure was used: Firstly, the consumption premium, Γ, was computed, which is required

[18] The inflation rate, π, was computed according to $CPI_{1994}(1+\pi)^7 = CPI_{2001}$, where CPI_{1994} and CPI_{2001} are the consumer price indeces in the years 1994 and 2001, respectively, as reported by UEPLAC (2001)

for compensating the individual for the total welfare loss associated with wage arrears

$$E[u(c_t + \Gamma)] = u(C^{SS}), \tag{22}$$

where C^{SS} denotes the non-stochastic steady state consumption in the economy without wage arrears (in *Table 3*, it is reported $\Delta^u C \equiv -\Gamma / C^{SS} \times 100\%$). Secondly, the cost of consumption fluctuations is measured as the difference between total welfare loss and aggregate consumption loss, $\Delta^u C - \Delta C, \%$. In the benchmark case, the cost of consumption fluctuations appeared to be 0.958%.[19]

The next step is to investigate how the results depend on the value of the risk-aversion coefficient, γ. The inspection of the results in *Table 3* reveals some interesting tendencies. As agents become more risk averse, they increase their precautionary savings, which reduces the aggregate capital income, *RK*, and raises the aggregate labor income, Ω. The total effect of this on aggregate income and aggregate consumption is positive, i.e., the income and consumption losses reduce. In contrast, the cost of the consumption fluctuation goes up. The quantitative expression of the effects associated with variations in γ can be very significant. For instance, as γ increases from 1 to 10, precautionary savings rise from 2.936% to 38.961%; aggregate income and consumption losses are reduced from 15.120% and

[19] Lucas (1987) proposed measuring the cost of business cycle fluctuations by a consumption premium, which must be given to the agent in order to provide her with the same utility level as the one that would be derived from the expected consumption. In the same way, it could be measured the cost of consumption fluctuations by a consumption premium $\tilde{\Gamma}$ satisfying $E\left[u\left(c_t + \tilde{\Gamma}\right)\right] = u(E[c_t])$. Lilia and Serguei Maliar found that under this measure the cost of consumption fluctuations is slightly higher. For example, in the benchmark case, it is equal to 1.120%.

12.845% to 10.025% and 8.229%, respectively; and, the welfare loss from the consumption fluctuations rises from 0.958% to 5.894%.

The robustness of the model's predictions was studied with respect to changes in α, δ, d by setting these parameters to values that are standard in macroeconomic literature: $\alpha = 0.36$, $\delta = 0.96$, and $d = 0.1$. The effects of variations in the other parameters were also analyzed. Specifically, a 10% increase in d_q, a 10% decrease in ψ_q and a 10% increase in σ_q relative to their benchmark values. Overall, the qualitative implications of the model proved to be robust to these modifications (see the last six columns in *Table 3*). Changes in precautionary savings varied from 2.084 % to 3.708%. Losses of income varied from – 15.570% to – 18.666% and losses in consumption varied from –13.257% to – 16.264%. Welfare loss from the consumption fluctuations was in the range of – 0.522% and – 0.979%.

One shortcoming in the analysis is that the calibration procedure neglects the effects associated with permanent heterogeneity in skills (productivity) by assuming that, in the absence of wage arrears, all agents would earn the same wage, *W*. The assumption is necessary because of the data problem. Specifically, the data is available on effective not on contractual wages. In order to evaluate the extent, to which the results are spoiled by the skill heterogeneity, the model's predictions under two alternative calibration procedures were computed. One was to split the sample into groups by education and to weight the wages and wage arrears of agents by the coefficients that reflect wage differentials across the educational groups distinguished. The other was to adjust the wages and wage arrears of the agents according to the wage differentials across qualification-profession groups. The tendencies described in this section proved to be robust to these modifications as well; and the quantitative expression of the effects associated with wage arrears was comparable to that obtained under the benchmark calibration procedure.

Chapter 6

Conclusion

The phenomenon of wage arrears is an exceptional feature of transition economies. According to the model developed by Lilia and Serguei Maliar this phenomenon can be addressed in the context of the neoclassical growth model, which is used for studying economic issues relevant to developed market economies. In Maliar's model the effect of wage arrears on the individual consumption-saving behavior is considered as idiosyncratic shocks to productivity. Two types of costs associated with wage arrears are distinguished. First, there is a reduction in effective wages because of the depreciation of wage arrears. Second, there is a welfare loss due to consumption fluctuations. The calibration of the model proved that these costs are substantial: consumption falls by 8%-16%, and welfare loss from idiosyncratic uncertainty is equivalent to an additional consumption loss of 1%-6%.

In the modeled economy, wage arrears are a survival strategy of the firms that incur losses during the transition process. The firms that were unable to perform well under new conditions of a developing market economy put their problems on the shoulders of their workers. As a result, inefficient enterprises were able to survive for a long period of time. So it can be considered that losses are much more significant than that calculated from the Maliar's model. Actually, it is very difficult (if possible) to evaluate the losses from the existence of a loss making enterprise.

So in the work there were evaluated some losses incurred by wage arrears. Although, the list of loss-points considered in the work is not complete the evaluation shows the importance of the problem. It is necessary to note that after some declining tendencies in the amount of wage arrears payment delays in wages again become to increase. So despite the "obvious" fact that wage arrears are harmful for Ukrainian economy, decision-makers go on solving their current problems by creating problems in the next period.

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Appendix A

The solution of the model and the methodology of computation of the non-stochastic steady state were suggested by Lilia and Serguei Maliar. To solve for the individual asset demand function, there was used an algorithm that computes a solution to the Euler equation (14) on a grid of prespecified points. Wage arrears were restricted to belong to the interval $q \in [\underline{q}, \overline{q}]$ and the interval was split into H equally-spaced points $\{q_1, \ldots, q_H\}$. The next step is computation of transitional probabilities associated with the autoregressive process for wage arrears (4) (i.e., the Markov chain). For each state $q \in \{q_1, \ldots, q_H\}$, the asset demand function is parameterized by a function of the currently available resources z. The grid for the resources consists of M equally spaced points in the range $[\underline{z}, \overline{z}]$. The minimum of resources, $\underline{z}$, is achieved when the agent has no asset holdings and receives the minimal possible labor income $\underline{\omega}$ (i.e., faces the maximum possible increase in wage arrears, from q1 to qH). The maximum of resources is $\overline{z} = \overline{\omega} + (1 + R)\overline{a}$, where $\overline{a}$ is the maximum sustainable capital stock (i.e., the solution to $F(\overline{a}) = d\overline{a}$), and $\overline{\omega}$ is the maximal possible labor income (i.e., the maximum possible decrease in wage arrears, from qH to q1). In particular, the construction implies that the individual asset holdings are restricted to be in the range $[0, \overline{a}]$. The baseline parameterization is H = 5 and M = 100. To evaluate the asset function outside the grid, Lilia and Serguei Maliar use cubic polynomial interpolation.

The algorithm, which iterates on the Euler equation, is employed. By substituting consumption from the Euler equation (14) in budget constraint (10), it is obtained

$$\hat{a}' \leq z - \left[\delta \sum_{q \in \{q_1,\ldots,q_H\}} \frac{(1+R)\Pr ob(q' \mid q)}{\left(A(z,q)(1+R) + \omega' - A(A(z,q),q')\right)^{\gamma}} \right]^{-1/\gamma},$$

(23)

where $\omega' = q(1-d_q) + W - q'$.

Consequently the following iterative procedure was implemented:

Step1. Fix some asset function on the grid, $A(z,q)$.

Step2. Use the assumed decision rule for assets to calculate the right side of the Euler equation (23) in each point on the grid. The left side of the Euler equation will be the new asset function, $\tilde{A}(z,q)$.

Step3. Compute the asset function for the next iteration $\tilde{\tilde{A}}(z,q)$ by using the updating:

$$\tilde{\tilde{A}}(z,q) = \eta \tilde{A}(z,q) + (1-\eta) A(z,q), \quad \eta \in (0,1].$$

For each point, such that $\tilde{\tilde{A}}(z,q)$ does not belong to $[0, \bar{z}]$, set $\tilde{\tilde{A}}(z,q)$ equal to the corresponding boundary value.

Iterate on Steps 1-3 until the fixed point is achieved with a given degree of precision, $\left\| \tilde{\tilde{A}}(z,q) - A(z,q) \right\| < 10^{-10}$, where $\|\cdot\|$ is the L2 distance.

Lilia and Serguei Maliar solve for the interest rate and wage corresponding to a given asset function $A(z,q)$ by computing an invariant probability distribution of the total resources and wage arrears, $\Pr ob(z,q)$, as described in Rios-Rull (1999):

$$\Pr ob(z',q') = \sum_{q \in \{q_1,\ldots,q_H\}} \Pr ob\left(A^{-1}\left(\hat{a}',q\right),q\right) \cdot \Pr ob(q' | q),$$

$$\hat{a}' = \frac{z' - \left(q\left(1-d_q\right) + W - q'\right)}{1+R},$$

where $A^{-1}\left(\hat{a}',q\right) = \left\{z, \hat{a}' = A(z,q)\right\}$ is the inverse of the asset demand function $A(z,q)$.

Finally to solve for the equilibrium fixed-point interest rate (the stochastic steady state), it was used a bisectional method proposed in Aiyagari (1994).

Appendix B

Alternative approach

Current method of calibration has a drawback. In the data, arrears are always non-negative and we have a lot of zeros. In the main part the process of arrears is described as

$$q_{t+1}^{s} = q_{t}^{s}(1-\hat{d}_{q}) + \hat{\omega} - \omega_{t}^{s} ,$$

where ω_t^s is drawn from a truncated Normal distribution,

$\omega_t^s \sim N(E(\hat{\omega}_t^s)), \mathrm{var}(\hat{\omega}_t^s))$ and ω_t^s >0. This approximation of wage arrears can be quite far ago from the true distribution of arrears. For better approximation the method described below was developed.

The calibration for wage arrears

The process for wage arrears of each individual *s* is:

$$q_{t+1}^{s} = \left(q_{t}^{s}\left(1-d_{q}\right)+\varpi^{s} n_{t}^{s}\right)\left(1-z_{t}^{s}\right) \qquad (*)$$

where ϖ^s is the wage of agent *s*, and n_t^s is working hours of agent *s*. In the model, agents supply all their labor endowment to the market, so that n_t^s = 1. Important to note that the wage ϖ^s is the wage that the individual *s* would receive at *t* in the absence of arrears. However, because of arrears, the individual receives only a fraction z_t^s of ϖ^s.

To compute the aggregate (average) level of arrears in the economy, we take the expectation of (*) and use the fact that q_t^s and z_t^s are uncorrelated to get

$$q = \left(q\left(1-d_{q}\right)+\varpi\right)\left(1-z\right).$$

If n_t^s =1, the wage is $\varpi = (1-\alpha)y$ and are therefore, we have

$$d_{q} = 1 - \frac{1}{1-z} + (1-\alpha)\frac{y}{q} \qquad (**)$$

The ratio $\frac{y}{q}$ can be computed from the data. Thus, if z is specified we can identify the depreciation rate of arrears d_q.

The next objective is to estimate equation (*) with the individual data. We consider two alternatives for estimation:

Process 1: z_t^s is drawn from a uniform distribution $[a,b] \subset [0,1]$,

Process 2: z_t^s is drawn from a Normal distribution $z_t^s \sim N(\mu, \sigma^2)$.

The processes 1 and 2 imply that, in each period, the agent receives a random fraction of the current debt, drawn from a uniform and Normal distributions, respectively.

For computation, it is necessary to renormalize the arrears by the wages, so $\tilde{q}_t^s = \frac{q_t^s}{\omega^s}$ and estimate the following process

$$\tilde{q}_{t+1}^s = (\tilde{q}_t^s (1 - d_q) + 1)(1 - z_t^s), \qquad (***)$$

For computation, we should use a simulated method of moments type of technique.

The algorithm is the following:

1) Take the empirical data on the arrears and construct the empirical distribution of arrears.

2) Fix the parameters in the process for z_t^s (such as μ and σ if z_t^s is drawn from a Normal distribution or the bounds *a* and *b* in the case of the uniform distribution), compute the parameter d_q from (*) and run a long simulation to generate the sequence for arrears. Use simulated data, to construct the simulated distribution of arrears and compare to the empirical distribution of arrears constructed in the Step 1.

3) Iterate on the parameters μ and σ until the simulated and empirical distributions are the same.

As a measure of closeness of simulated and empirical distributions, I have used their moments.

I have written a MATLAB program that solves the task described above with the use of "fsolve" function. "Fsolve" minimizes the loss function

$$f(z) = \left[\alpha(mean - mean(z))^2 + (1-\alpha)(std - std(z))^2\right];$$

where *mean* – mean of actual distribution;

mean(z) – mean of simulated distribution;

std – standard deviation of actual distribution;

std(z) –standard deviation of simulated distribution;

α - value of mean in the loss function, $0 < \alpha < 1$.

The MATLAB programs are attached in Appendix C.

Appendix C

The program defines loss function and simulates distribution

```
function F= minimiz(z)
global r c m1 sd1 s P WA q;
        F= [(((m1-m(z))^2)*0.8)+(((sd1-standdiv(z))^2)*0.2)];

function sd2 = standdiv(z)
global r c m1 sd1 s P WA q;
e = s*q;
d = P - (z/(1-z)); % depretiation rate
WA1 = (WA.*(1-d)+1)*(1-z);
Q = (WA.*(1-d)+1); % WA+1 = (WA.*(1-d)+1)*(1-z)+ (WA.*(1-d)+1)*(random
element)
Q1 = zeros(r,c); % calculates Q1 = (WA.*(1-d)+1)*(random element)
for i = 1:r,
   v = Q(i,:);
   x = e(i,:);
   b = x.*v;
   Q1(i,:)=b;
   b = zeros(1,1);
end
WAsim = zeros(r,c); % calculates WA+1 = (WA.*(1-d)+1)*(1-z)+ (WA.*(1-
d)+1)*(random element)
for i = 1:c,
   g = Q1(:,i);
   b = g+WA1;
   WAsim(:,i)=b;
end
sdWAsim = std(WAsim);    % standard deviation by columns
sd2 = mean(sdWAsim);     % average standard deviation
% inputs z and q
% other inputs r, c, m1, sd1, s, P.

function m2 = m(z)
global r c m1 sd1 s P WA q;
e= s*q;
d = P - (z/(1-z)); % depretiation rate
WA1 = (WA.*(1-d)+1)*(1-z);
Q = (WA.*(1-d)+1); % WA+1 = (WA.*(1-d)+1)*(1-z)+ (WA.*(1-d)+1)*(random
element)
Q1 = zeros(r,c); % calculates Q1 = (WA.*(1-d)+1)*(random element)
for i = 1:r,
   v = Q(i,:);
   x = e(i,:);
   b = x.*v;
```

```
    Q1(i,:)=b;
    b = zeros(1,1);
end
WAsim = zeros(r,c); % calculates WA+1 = (WA.*(1-d)+1)*(1-z)+ (WA.*(1-d)+1)*(random element)
for i = 1:c,
    g = Q1(:,i);
    b = g+WA1;
    WAsim(:,i)=b;
end
mWAsim = mean(WAsim);     % mean by columns
m2 = mean(mWAsim);        % total mean
% inputs z and q
% other inputs r, c, m1, sd1, s, P.
```

The program iterates with moments and minimizes the loss function (approximates simulated distribution to the empirical one).

```
LL=1;
while LL<20;% the cycle is defined for obtaining a sample of outcomes
r = 676;   % rows (in case of WA0 distribution r = 548) otherwise r = 464
c = 1000; % colomns
WA=WANEW0; % in case of other distribution
m1 = mean(WA); % mean of wage arrears distribution
sd1 = std(WA); % standard deviation of wage arrears distrtibution
s = rand(r,c); % random draw with Normally distributed elements
P = 2.202416578; % an indicator P = (1-a)*(y/q)
global r c m1 sd1 s P WA q;
q=1.05;
z=0.687;
j=1;
%sd = standdiv(z,q,s,P,WA,r,c);% standard deviation of simulated distribution (standiv.m)
%mm = m(z,q,s,P,WA,r,c);% mean of simulated distribution (M.m)
%ww = minimiz(z)
while q <1.15;
X0 = [z];
[T,fval] = fsolve(@minimiz,X0,optimset('Display','iter'))
HH(1,j)=fval;
HH(2,j)=T;
HH(3,j)=q;
j=j+1;
q=q+0.001;
end
[ff,i]= min(HH(1,:));% minimum value from the set of values
Z = HH(2,i);% the value of Z that corresponds to the minimum value of the objective function
sigma = HH(3,i); %the value of sigma that corresponds to the minimum value of the objective function
d = P - (Z/(1-Z));
if d<0 while d<0;% the cycle is defined to select outcomes that satisfy positive depretiation rate
    HH(1,i) = 10000;
    [ff,i]= min(HH(1,:));% minimum value from the set of values
    Z = HH(2,i);% the value of Z that corresponds to the minimum value of the objective function
    sigma = HH(3,i); %the value of sigma that corresponds to the minimum value of the objective function
    d = P - (Z/(1-Z));
  end
Z = HH(2,i);% the value of Z that corresponds to the minimum value of the objective function
sigma = HH(3,i); %the value of sigma that corresponds to the minimum value of the objective function
d = P - (Z/(1-Z));
end
```

```
LL=LL+1;
j=1;
GG(1,j)=Z;
GG(2,j)=sigma;
GG(3,j)=d;
j=j+1;
end
avZ = mean(GG(1,:))% average of the repeated outcomes of Z
avsigma = mean(GG(2,:))% average of the repeated outcomes of sigma
```