

IS THERE EQUILIBRIUM IN THE
TRAVELERS' DILEMMA WITH
HETEROGENEOUS AGENTS?

by

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Abstract

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The thesis investigates equilibrium in the 'Travelers' Dilemma with heterogeneous agents. Proposed risk sensitivity and cost approaches allow explaining the deviation of real player from the theoretical Nash Equilibrium. The investigation applies risk-dependent utility function and tests player's strategies with developed Matlab simulation program for Travelers' Dilemma.

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Chapter 1

INTRODUCTION

The Travelers' Dilemma (further denoted as TD) is described in Basu (1994): "Two travelers returning home from a remote island, where they bought identical antiques (or, rather, what the local tribal chief, while choking on suppressed laughter, described as "antiques"), discover that the airline has managed to smash these, as airlines generally do. The airline manager who is described by his juniors as a "corporate whiz," by which they mean a "man of low cunning," assures the passengers of adequate compensation. But since he does not know the cost of the antique, he offers the following scheme.

Each of the two players has to write down a piece of paper the cost of the antique. This can be any value between 2 units of money and 100 units. Denote the number chosen by traveler i by n_i . If both write the same number, that is, $n_i = n_j$, then it is reasonable to assume that they are telling the truth (so argues the manager) and so each of these travelers will be paid n_i (or n_j) units of money.

If traveler i writes a larger number than the other (i.e., $n_i > n_j$), then it is reasonable to assume (so it seems to the manager) that j is being honest and i is lying. In that case the manager will treat the lower number, that is, n_j , as the real cost and will pay traveler i the sum of $n_j - 2$ and pay j the sum of $n_j + 2$.

Traveler i is paid 2 units less as penalty for lying and j is paid 2 units more as reward for being so honest in relation to the other traveler.

Given that each traveler or player wants to maximize his payoff (or compensation) what outcome should one expect to see in the above game? In other words, which pair of strategies, (n_i, n_j) , will be chosen by the players?"

Best response for each player is to play amount of other player minus one, hence, she can achieve bonus. This logic leads to playing lowest game bound and exactly amount of other player (lowest game bound). On the basis of game theory, both players should choose minimal amounts of possible. This outcome gives minimal total utility of the pair.

Basu claimed that it could be better for both players to make assumption of irrationality of counterpart, or at least, expectation of irrationality. Thinking that there is probability that other player can choose relatively large number leads to the transforming the Nash Equilibrium $[\min, \min]$ into the Bayesian-Nash Equilibrium. However, Bayesian-Nash equilibrium is broken if such beliefs are not consistent with a reality.

Monica Capra (1998) made experiment by which strategies were in the range $[80; 200]$ cents, penalty and bonus were 80. Game theory predicts Nash equilibrium $(80, 80)$. But players' real decision was in average 180. Increase of bonus from 5 to 80 cents, led to the average outcome decrease to 120. Higher award led to the outcome closer to the Nash Equilibrium. This experiment was continued by allowing repetition of the game. Results showed that when award was high, players were moving to the NE, while games with small bonus converged out of NE into the upper limit. These results show that

amount of bonus influence the decision. But there is trade off between bonus maximal payoff and risk. In this research the influence of risk into the decision making process and equilibrium is investigated.

Another experiment, Rubinstein (2006), with 2985 observations showed the same problem: the theoretical NE is not played and in real life distribution of outcomes it is rationally to deviate from theoretical NE.

In case when each agent knows that counterpart is rational (can precede information) there is no support of forming Bayesian-Nash Equilibrium. In the experiment performed by Tilman Becker took part the 51 members of the Game Theory Society. They played in [2; 100] range. The player with highest average pay off received a real money award. Each gamer was supposed to be familiar with Nash concept, and was rational with knowledge that others players are rational as well. But only 4% played NE. While 46% of players chose a number in range 95-99. And 20% chose 100.

The real life example of the general Travelers' Dilemma is a difficult issue, but Basu claimed that in general it could be connected to the arm race. We can expand this explanation, that Traveler's Dilemma approach can be considered as simplification of arm race. Expenditure limits are determined by upper and under bounds. Maximal amount which country may spend on weapon under budget constrained is upper bound. Under bound is minimal expenditure on army. This limit determined by both intra country determinans and general international situation except potential enemy. Military expenditure decrease consumption. Upper bound in military expenditure represent under bound in consumption. And vise-a-versa: under bound of military expenditure reflect

upper bound in consumption by certain nation. In our TD it reflects in range [2, 100].

Utility of this game distributes by following scheme. If two equal nations have same military expenditure, then same amount of resources is remained for the consumption and they have same utility. If one nation decrease initial planned consumption, that is it increase expenditure on weapon (in order to make one more innovation, in our model it decreasing consumption by 1 unit). It leads both to greater bargaining power in international relations, and better competitiveness of the country's weapon in the third countries markets. Hence, this country obtains some benefit (in our dilemma it equal to 2 units). Game theory predicts behavior of the countries as follows. Both think to consume 100 and remain minimum sum to the weapon development. Then one country thinks to develop one more gun in order to obtain 2 units of benefits and have consumption 101. Other country understands such possibility and decrease consumption by two units in order to have more developed gun. Then first country decreases its consumption and so on..... They end up in NE $\{2, 2\}$. Other real life example of this Dilemma is Bertrand Competition on discrete set.

In this research we will introduce the cost of thinking approach to the Travelers Dilemma. Risk dependent utility function will be introduced as well. These approaches haven't been used yet for this Dilemma.

Chapter 2

LITERATURE REVIEW

In this literature review I'm going to consider papers about rationality of agents, starting from those of them, which consider pure game theory rationality as a not best behavior in all real situations, particularly in Traveling Dilemma problem. Review of economical papers about bounded rationality and economic applications of it will set up the problem. Then will be brief revision of Psychological papers which are concentrated on rationality, and other types of decision making process. Psychological knowledge will be reconsidered with analogue to the papers focused on the Bayesian Nash equilibrium. Finally, literature review considers works about technique of finding equilibrium, because these techniques will allow us to find new type of equilibrium in static symmetric in terms of utility games with complete information, but heterogeneous agents in terms of rationality or in terms of costs of thinking

Best response for each player is to play amount of other player minus one, hence, she can achieve bonus (and exactly amount of other player, if it equals to low game limit). On the basis of game theory, using backward induction, both players should choose minimal amounts of possible. This outcome gives minimal total utility of the pair. Basu proposed possibilities of solving the paradox. Using the curb sets conception on continuum interval (they self-

enforce to play in interval minimal limit of which higher than minimal limit of whole game), which is not our case, because they should choose integers. Basu claimed that it could be better for both players to make assumption of irrationality of counterpart, or at least, expectation of irrationality. However, thinking that there is probability that other player can choose relatively large number leads to breaking of the Nash Equilibrium.

Empirical data confirm the paradox and possibility of making better decision than just choosing Nash Equilibrium strategies. Ariel Rubinstein in 2002 performed an experiment in the range [180; 300] and bonus or punishment is 5. The distribution of real decisions was:

180	181–294	295	296–298	299	300
13%	15%	5%	3%	9%	56%

That is just 13% played NE, while more than one half played other extreme, 300. Rubinstein made hypotheses to explain such distribution. Answer 300 is likely to be intuitive answer. Answers in 295–299 range are seemed to be done due to strategic reasoning, because these answers can give greater payoff (due to bonus). Answers in the range 181–294 are assigned as random. And 180, which is by theory is NE could be done due to prior acquaintance with game theory prediction. Rubinstein concluded that different agents are heterogeneous in ability to process information. He supported this idea by the time spent to the response. Random (in range 181–294) and intuitive (300) choices have been done relatively faster than rational reasoning outcomes (in

range 295-299) – 70 seconds vs. 96 in average. Hence, heterogeneity should be introduced in finding of new best response and Equilibrium.

Let denote expected pay off of plying strategy “a” as $U(a)$.

$$U(180) = (1-.13)(180+5) + .13*180 = 184.35;$$

Let now take deviation for instance to 294, then pay off will be (and assuming even worst case when all persons in the range 181-294 claim 181):

$$U(294) = .13(180-5) + .15(181-5) + .05(294+5) + .03(294+5) + .09(294+5) + .56(294+5) = 240.51$$

On this base, Rubinstein (2006) concluded: “The players who chose 180 are probably aware of the game theoretical prediction. On average, they would do badly playing against a player chosen randomly from the respondents. These players can claim to be the “victims” of game theory. The subjects whose answers were in the range 295–299 clearly exhibit strategic reasoning. The answer 300 seems to be an instinctive response in this context and the responses in the range 181–294 appear to be the result of random choice.” Hence, population can be divided on the three groups: strategic reasoning agents (I would like to include to this group not only those who have chosen 295-299, but victims of game theory as well), instinctive agents, and randomly behaved agents. This data shows heterogeneity of agents, which is more close to real life than assumption of total rationality. But here arises a question about existence of equilibrium in games of heterogeneous in term of rationality agents.

The possibility of the equilibrium in particular game with heterogeneous agents has been showed by Haltiwanger and Waldman (1985). Due to heterogeneity in information processing they considered theoretical population with those who can process information without restrictions, and limited agents. Hence, a model with two types of agents “sophisticated” and “naïve” has been used. Simplification of the model was that. Pay off was dependent only on number of other people chose same strategy. In first case more players act similarly, less pay off of each. In finance it can reflect the case of opening the banking office in particular area. Analysis showed that such game structure leads to bias in rational agents’ decision away from expected action of irrational players. This bias is due to the fact that naïve agents’ actions have a certain bias in comparison to total rationality assumption. The bias of irrational agents decisions lead to new best response of rational agents, so they achieve new equilibrium.

Second case considered situation, when acting same strategies more beneficial. In case of synergetic dependence, the bias of rational agents is inward to the expected action of irrational. Hence, we achieve new equilibrium even in this case. So, in this specific game equilibrium does exist. Moreover, in both cases equilibrium is biased in comparison to Nash Equilibrium under total rationality assumption.

Empirical testing of bounded rationality influence on aggregate outcome was done by Fehr and Tyran (2007). In this work adjustment of nominal prices after a fully anticipated monetary shock has been tested. By theory if all agents

are fully rational, then anticipated monetary shock will have no effect on real variables. In fact authors showed that influence on real variables was significant. Hence, standard macroeconomics models with all rational agents are not correct here. Therefore, binding rationality should be incorporated.

The psychology makes fact about binding rationality more close to the real life. The research by Kahneman (2003) shows that from the psychological point of view, decisions that seem to be non-rational have deeper psychological roots. Such decisions are made not because of less information processing abilities, but rather due to decision making conditions and type of an agent. Under time pressure decisions are more likely to be intuitive or heuristic. Nevertheless, there are agents that behave intuitively more often in any conditions. And there are different mechanisms of heuristic decisions: adjustment, available heuristic (when decision depends on observed cases in past). However it is noticeable that by playing “intuitive strategy” one can have a deeper roots of thinking that just payoff In our model types of agents will be dependent on risk sensitivity, and ability to process information (through cost of thinking approach).

Chapter 3

METHODOLOGY

The goal of this work is to find theory best explanation to the strategy played by real players and define what equilibrium appears here. Explanation introduces best response and equilibrium, if any. The first stage is to introduce best response. Players think as follows:

- 1) Wants to maximize their payoff (basic, as in Nash concept);
- 2) Aimed to get bonus if possible, so they have incentive to decrease bid (as in Nash concept);
- 3) Don't want to undercut their possible profit, so they have incentive to increase bid; they are estimating opportunity costs;
- 4) Each step of thinking has the costs, so player thinks till the marginal cost of thinking is less than marginal benefit of such thinking (optional);
- 5) They assume that other player is not going to undercut the possible profit;
- 6) They know that other players have the same thinking approach (4);
- 7) The utility depends not only on expected payoff, but on estimated risk as well. (As a proxy of risk we take standard deviation of expected average counterpart strategy).

Last points are new in for the Travelers' Dilemma

If bonus/punishment is small enough then their opportunity costs of lowering the bid are higher than their possible bonus. As a first stage, I'm considering simple case: we game versus one of two gamers – intuitive and rational. Probability is 50/50. Intuitive always play upper bound, rational has same logic as we are. What should we play? My observation about data mentioned: “strategic” decisions are within one amount of bonus under upper bound, which is in Rubinstein's data 295-299 (when upper bound is 300 and bonus is 5) that is because these bids can give higher payoff than intuitive one.

This definition of best response and Nash Equilibrium may be similar to Bayesian Nash Equilibrium under bounded rationality. However Becker's experiment assures us that all agents are rational (they are the Game Theory Society members). Hence, their “irrational” actions have some deeper economic background. On other hand, all participants knew that counterpart is rational agent as well. That is here in their incentives exist more concerns than just Nash reasoning.

Our new best response can be described in following example related to TD. As in simple Nash reasoning we start with 100. And think that 99 can always give us greater payoff. Then 98 is better strategy against 99. But such thinking will lead us to NE (2, 2). Hence, at some point G (that can be 100, 99 etc.) the agent thinks: there is no evidence that counterpart will play G for sure. That is there is probability of counterpart strategy 100, $P(100)$, probability of counterpart strategy 99, $P(99)$,
.....,

some probability of counterpart strategy G , $P(G)$,

probability of counterpart strategy $G-1$, $P(G-1)$,

....,

probability of counterpart strategy 3, $P(3)$,

probability of counterpart strategy 2, $P(2)$.

And cumulative function $F(G) = \sum_j P(\text{strategy } j \geq G)$,

At this point we don't make any assumption about these probabilities, but

$0 \leq P(\text{strategy } j) \leq 1$, natural assumption that probability can't be negative or greater than 1.

$\sum_j P(\text{strategy } j) = 1$ (sum of all probabilities is equal to 1)

Agent can either decrease the bid or stay at the point. Decreasing the bid by one point will

- decrease maximal possible pay off by 1, with probability $1 - F(G+1)$;
- increase this pay off with probability $P(G)$ by 1;
- stay at the same payoff with probability $F(G)$.

The equilibrium appears if

$P(G) < 1 - F(G+1)$.

It was about monetary pay off. In our model utility of the player depends on the monetary pay off and the number of iterations made.

Each step of thinking process by agent I costs "ci"

Cost of seeing tendency (fixed): c_i

Total costs of thinking:

$C_i = (100 - s_i) * c_i$

where sir is strategy of player i , who is rational

Or

$$C_i = (100 - sir') * c_i + c_{fi}$$

where sir' is stage of thinking when rational player i saw a tendency and switched immediately to the NE

If $R < c_i$, that is if reward less than cost of step of thinking, the player don't think. Hence, he became intuitive player. so his strategy is s_{ii} . This strategy may be any number in whole range $[2, 100]$ or $random[l, u]$. More likely (and for simplicity), $s_{ii} = u$. Intuitive players will play cooperative equilibrium $(100, 100)$.

In our model we use not only linear costs of thinking approach. If costs of thinking are greater than 1, the player will spend 1 unit for thinking costs and to calculate necessity of deviation from 100 to 99. He/she in case of success will have a bonus 2, so total payoff will be $99 + 2 - costs = 101 - costs < 100$. So, those who have high costs of thinking (greater than 1) will play 100 as a strategy, because in case of thinking they will spend more than can obtain as a result of deviation. Hence, only if costs of thinking are less than one, the agent will think as rational one. In case of evidence that all other players are play NE, our player will play NE $(2, 2)$ as well. Moreover each following step will consume less cost, because of learning influence and because of possibility to see a tendency.

Herewith we should make assumption about other approach of measuring thinking costs. Each following step of thinking will require less and less efforts with the constant (for simplicity) discounting rate. If this rate is equal

to the marginal cost of thinking, the total cost of thinking till the strategy j (taking into account the cumulative effect) will be:

$$Cost = mc + mc^2 + mc^3 + \dots + mc^{u-j} \quad (1)$$

Where,

mc – marginal costs of thinking (costs of each step of thinking);

j – strategy actually played;

u – starting point of thinking (in our case it is upper bound, 100).

It is easy to show that if agent don't think at all, he/she would play 99, the cost of thinking will be exactly mc .

Def: Best response is a set of actions (in TD it is only one action) that:

1. Gives maximal utility that depends on

- Beliefs about other players' (only one player in TD from the set of players) actions;
- risk, which is measured as standard deviation of the possible payoff;
- some parameter c_i and c_{if} of cost of thinking

The utility function of the players is:

$$U(m, C_i) = m - r \cdot \sigma - C_i \quad (1)$$

Where,

m – monetary pay off

r – coefficient that characterizes risk aversion of the player. Higher r will lead to higher risk aversion, and if r is equal to 0, the person becomes risk neutral;

σ – standard deviation of the pay off;

C_i – Cost of thinking (described earlier)

Def: Equilibrium is an outcome of the game [TD] when:

1. all rational players adopt Best response, described earlier,
2. all irrational players play their predicted actions (stable over time)
3. Beliefs of rational players are stable over time
4. If game is played one more time, then rational player wouldn't deviate from previous best response (because they have same beliefs, distribution and coefficients).

In experimental part we will estimate utility of players from the real data. We'll do it by combining Each Players' action with each counterpart. But in difference of Becker (2005) explanation we will check each observation of belief about level of accuracy. Types of agents will be not as given in our model, but explained by the cost of thinking approach and expected risk approach. It is needed due to following reason: More accurate beliefs distribution have been made by the player, less costly it was to think about the problem. This approach is needed to explain the choice of strictly dominated strategies made by several players.

For convenient using of data we introduce matrix "PLAYER_{np x 8}", where np is number of players. This matrix have both data exogenous data (e.g. second column – strategy, played) and endogenous of the model data (e.g. utility).

In general

PLAYER=[preE(payoff), strategy, mcosts, E(payoff), st.dev, risksens, utility, costs].

where

preE(payoff)_{np x 1} - vector (45x1) of payoff, expected by player before the game;

strategy_{np x 1} – vector (45x1) of players' strategies;

mcosts_{np x 1} – vector (45x1) of marginal costs *mc* of each player;

E(payoff)_{np x 1} – vector (45x1) of expected payoffs as a result of the game;

st.dev_{np x 1} – vector (45x1) of standard deviations as a result of the game;

risksens_{np x 1} – vector (45x1) of risk sensitivities of the players;

utility_{np x 1} – vector (45x1) of actual utilities as a result of the game;

costs_{np x 1} – vector (45x1) of costs of thinking of the players.

All endogenous columns will be described or obtained later, but in very beginning they are assigned as zero-vectors.

Matrices of beliefs (“fbeliefs_{np x u}”) and of strategies we take from the data (see description part).

We calculate matrix (PAYOFF, 3-dimantional matrix). First and second dimensions reflect the player strategy and player's counterpart strategy respectively. Third dimension assigns payoffs to the player (first array) and counterpart (second array). This matrix is obtained from the matrix of players' strategies, taking into account bonus. It is the core part of Travelers' Dilemma and calculation algorithm is taken from the set up of the Dilemma.

If strategy of players' *i* strategy (*n_i*) less than strategy of counterpart (*n_j*) then player *i* will have payoff (*n_i+2*), while player *j* – (*n_i-2*). In the case of equality both swill have *n_i=n_j*. Otherwise player *i* will have payoff (*n_j*) and player *j* – (*n_j+2*).

strategy_{np x 1}

bonus

=> PAYOFF_{np x np x 2} (2)

From this last matrix we calculated expected payoffs, of the players, and filled column vector “ $\text{payoff}_{np \times 1}$ ”. On other hand we calculated the variation of the payoff and standard deviation “ $\text{stdev}_{np \times 1}$ ”.

The vector “ $\text{risksens}_{np \times 1}$ ” is set exogenously, because riskaversion is external factor. We can only calibrate in some extend this vector, but we’ll do it later.

The vector “ $\text{mcosts}_{np \times 1}$ ” is set in similar way (we start from the situation when thinking is costless, so it is zero vector).

Vector “ $\text{costs}_{np \times 1}$ ” obtained from the $\text{mcosts}_{np \times 1}$ by the one of two described above methods (either linear or cost depreciating approach).

Given vectors of payoff, risksens and mcosts we can calculate actual utility of the players. Using formula (1),

$$\text{utility}_{i,1} = \text{payoff}_{i,1} - \text{stdev}_{i,1} * \text{risksens}_{i,1} - \text{costs}_{i,1} \quad (3)$$

In a similar way, but using beliefs of the strategies distribution, we calculate the matrix of expected utility for each player, given beliefs and matrix of the normal form representation of the game.

$\text{WEIGHTEDPAYOFFMATRIX}_{np \times u \times u}$ is a 3-dimensional matrix

1st dimension – the player (1,np) range;

2nd dimension – each strategy of the player;

3d dimension – each possible strategy of counterpart.

Each elements of this matrix obtained by using core ‘Travelers’ Dilemma set up (as was described earlier), but weighted on beliefs considering probability of all counterpart strategies.

From $WEIGHTEDPAYOFFMATRIX_{np \times u \times x \times u}$ we obtain $EPAYOFF_{np \times u}$ matrix which is simply expected payoff (the weighted sum of payoffs) of each player (1..np), if he/she play strategy (1..u).

On other hand from $WEIGHTEDPAYOFFMATRIX_{np \times u \times x \times u}$ we easily calculate the $STRVARIANCE_{np \times u}$ the matrix that assign expected variance for each player if he/she will play each possible strategy.

But in our utility function we use standard deviation rather than variance so the matrix of standard deviation $STDEVSTRATEGY_{np \times u}$ is a simple positive square root element-by element of corresponding variances from the $STRVARIANCE_{np \times u}$.

Since, this approach is connected with the data, more specific results will be provided in the next section.

Chapter 4

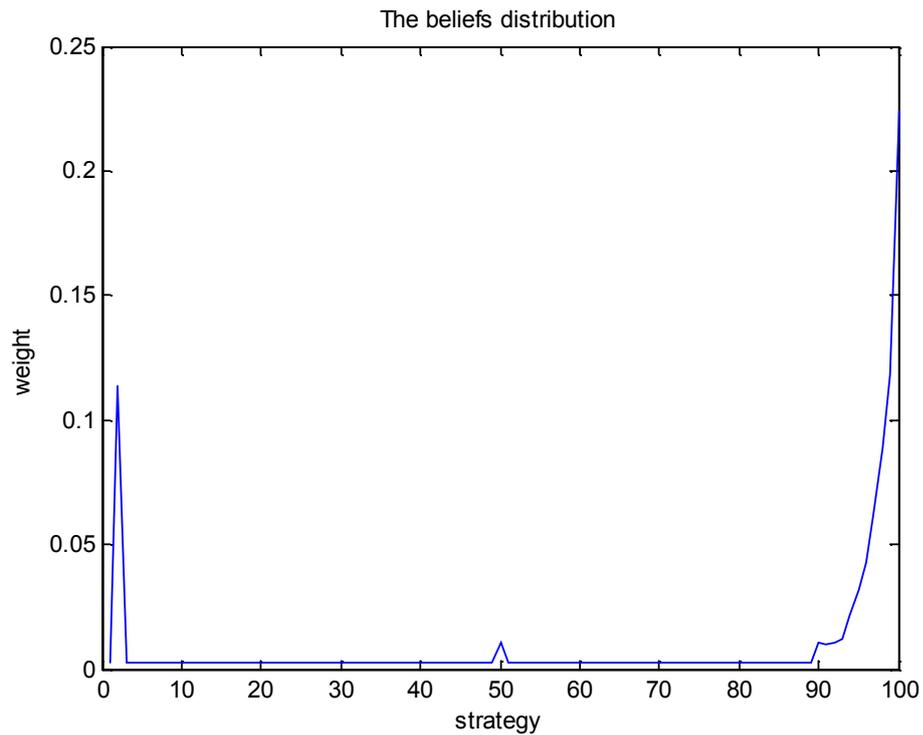
DATA DESCRIPTION

Our data is available for public data collected by Becker (2005) and published in the paper. Each agent is the member of the Game Theory Society. They were playing for the real money award (distributed by the special scheme and aimed to replicate conditions of the initial Dilemma). Number of observation participants in this experiment was 51. But only 47 submitted their beliefs and 45 played the pure strategy. For the sake of both simplification and accuracy, we will work only with 45 players. They played in the $[2, 100]$ limits, which is classical Basu's Travelers Dilemma. Minimum strategy played – 2 (3 observations). It is the first signal that standard NE (2) doesn't work here. The maximal strategy played – 100 (10 observations). It is the second surprising result, because 100 is strictly dominated strategy. All players are supposed to be deeply familiar with the Travelers Dilemma, and should have rational reasoning, since participants are game theorists, and participation in the game was optional. On other hand it can be played so widely, because it is cooperative equilibrium. But set up of the game was the constructed in a way that doesn't allow any cooperation. Mode was the 100 strategy. Second widely used strategy is the 98. In other hand data includes beliefs about distribution player players' strategies. Mode of beliefs was 100. Second widely used belief was 2 (with 18% weight). Since 3 assigned their beliefs as a strategy 2, and put

nothing for other strategy, and the total number of observations is not high enough, it drastically influenced on beliefs distribution.

Chart 1

The distribution of beliefs in the experiment (Becker, 2005)

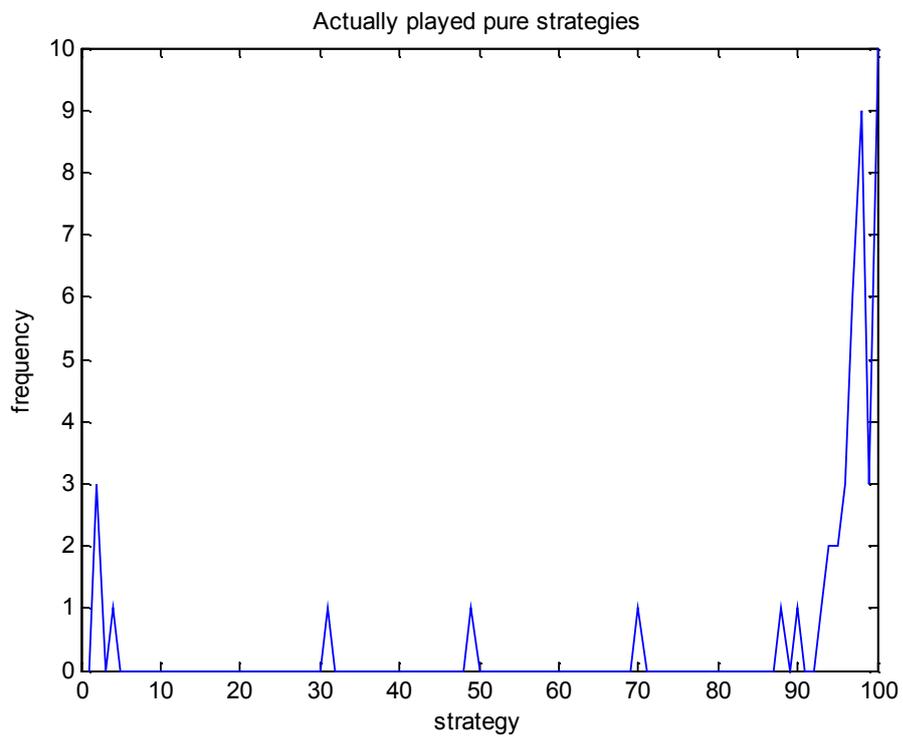


We recalculated the aggregate beliefs distribution of other 42 player. Because absence of precise beliefs distribution of these players we assign this distribution as being equal to all 42 players. This step doesn't lead to the losing of generality. On one hand it is seems to be harder task to explain diversity of players actions given same beliefs. On other hand we are going to explain this diversity by different parameters in social utility function. In fact as we seen real players played NE even less than prior beliefs. Drawback of the data is small enough of the observation and absence of the time variable

or answering or any other proxy that can be used explicitly as a measure of thinking costs.

Chart 2

The distribution of played pure strategies in the experiment (Becker, 2005)

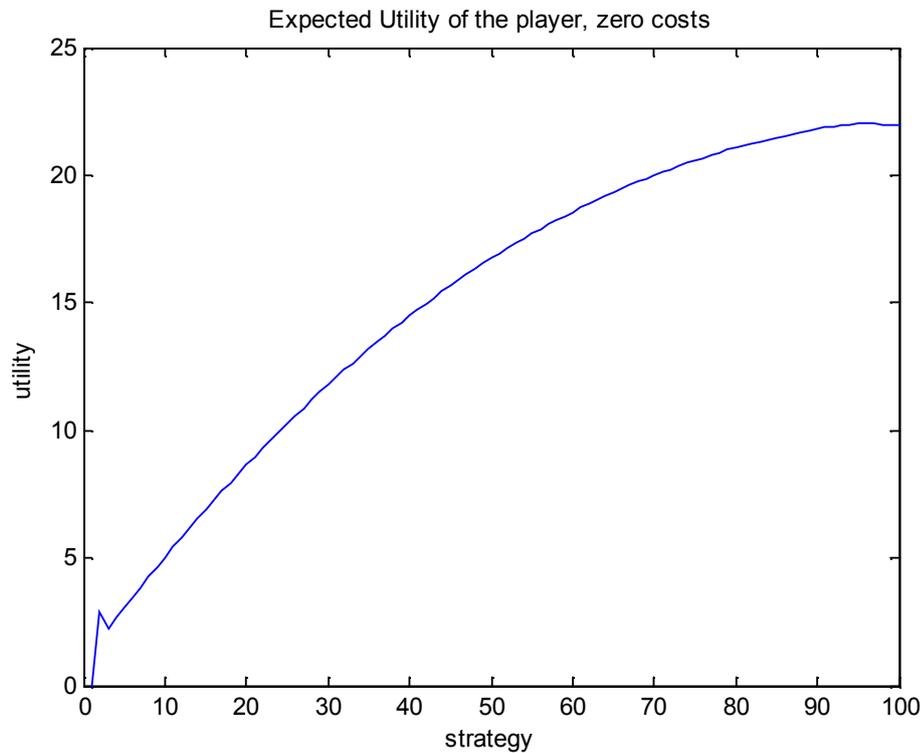


However this data have following additional drawbacks. The data or proxy considering risk aversion and costs of thinking is not presented here. Nevertheless, given beliefs and actions we will calibrate some possible values of these parameters.

RESULTS

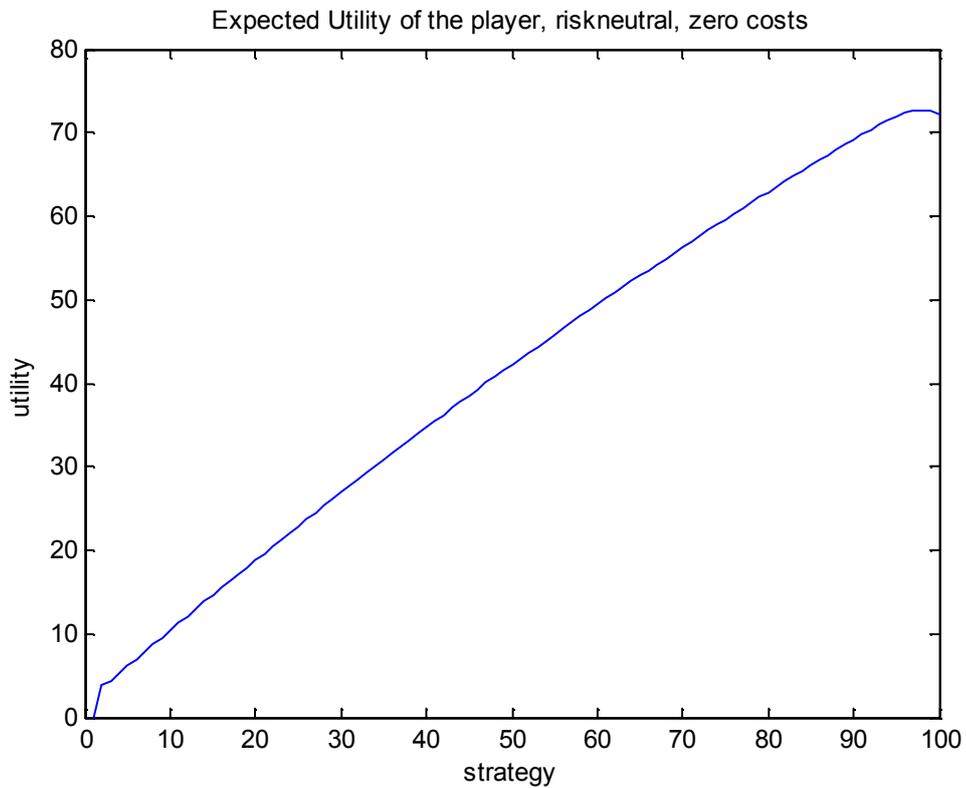
First and expected result is that given beliefs distribution expected payoff is in general increases with increasing of strategy. Substituting different risk sensitivity coefficients we achieved following result. If this parameter is greater than 1.377 even in absence of costs of thinking, but under aggregates beliefs of this experiment the strategy {99} becomes strictly dominated by strategy {100}. This fact shows that so called “cooperative strategy” can be implemented if utility of agents contains not only expected payoff, but risk and risk sensitivity factor (please, see Appendix B).

Chart 3



However, without taking cost of thinking into consideration we have double local maxima: {100} with utility 21,9715 and {97} with utility 22,0160. And strategy {97} is a global maximum. As it can be seen the difference of utility is not very high. Moreover, those who play 98 (instead of 100 or 99) should make to steps of backward induction reasoning in order to achieve this strategy.

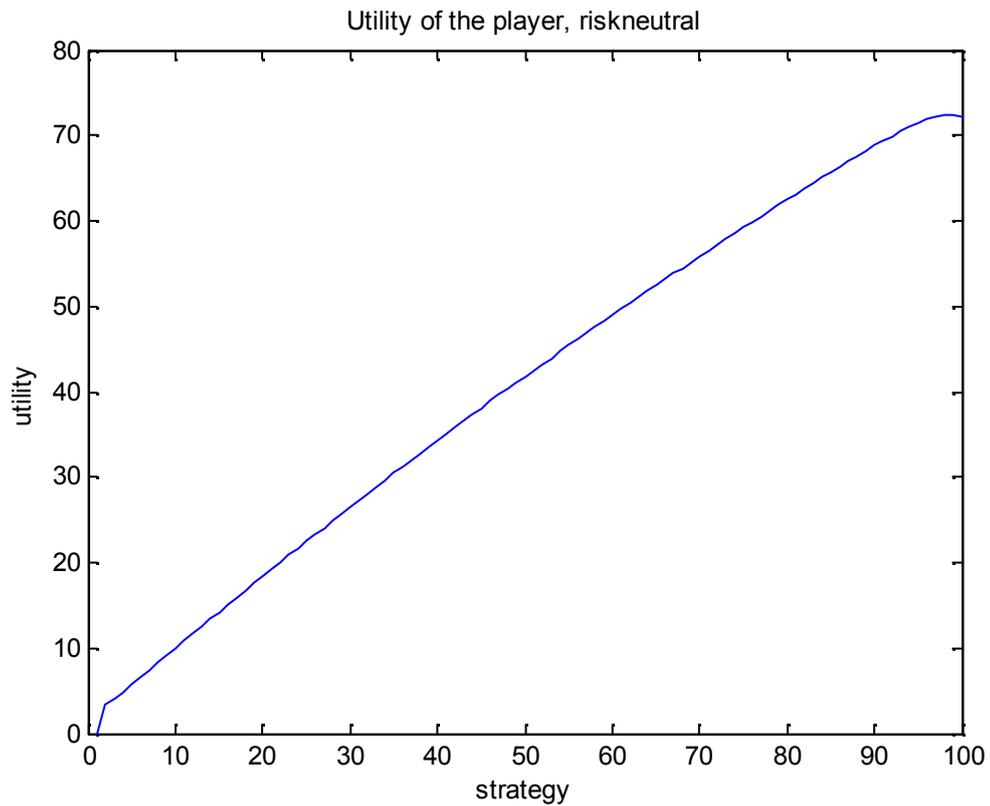
Chart



As can be see from Chart 5.2 and Chart 5.3 the adding of thinking cost parameter doesn't change the pattern drastically. Since each step of backward induction requires some efforts we can recalculate their utilities given marginal costs ($mc=.3$) and depreciation rate of costs 0.3. We

described it as the second method of cost calculation. This method gives total costs of thinking on the second step as $0.3+0.3^2$, which is 0.39.

Chart 5

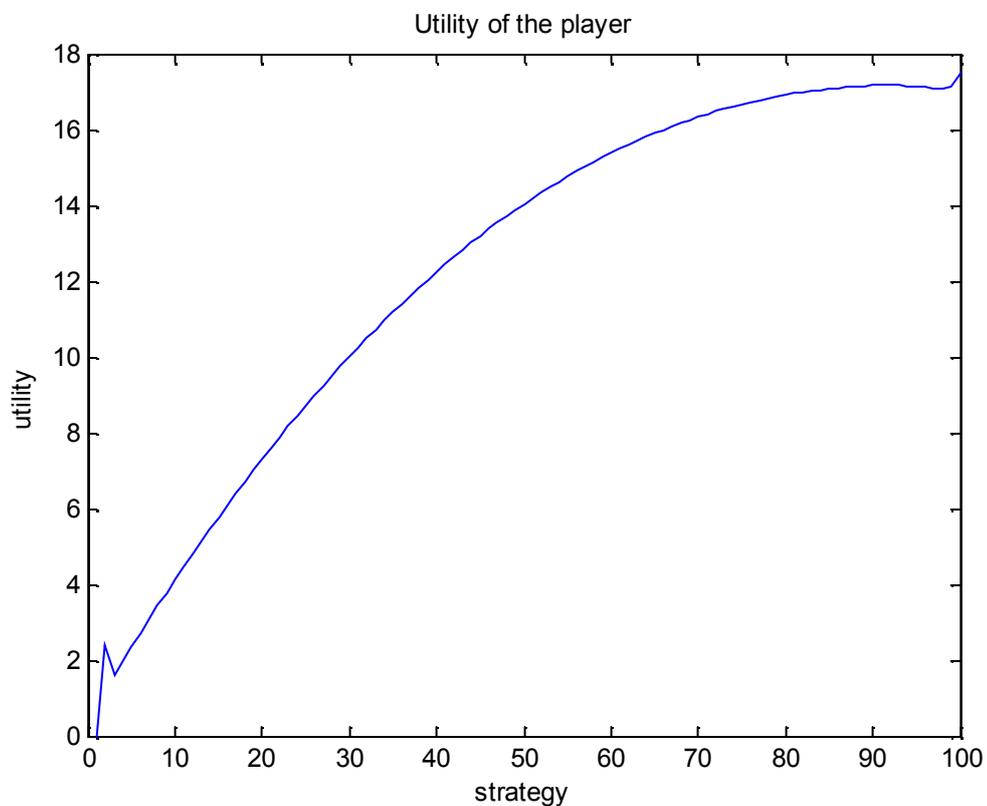


These method charges thinkers less then first method, which have $0.3+0.3=0.6$ as a cost of thinking. On other hand rate 0.3 is much less than 1, which is possible additional benefit form lowering the strategy played. On other hand the strategy {100} dominates {99} onany level of risk sensitivity higher than 1.377. As an example for all further calculation we take 15 as a parameter whenever risk sensitivity included inthe model and parameter 0.3 as mc of thinking.

The global minimum for this experiment is playing strategy {3}. On one hand this strategy is strictly dominated by {2} and gives zero payoff to the player, who play this strategy. On other hand this strategy is strictly dominates all strategies higher than 3. But since both beliefs distribution and actual distribution of the strategies gives to the strategies as {4}, {5} etc. quite low weight, player, who plays strategy {3} have incentive to deviate to higher strategy. Moreover, this strategy gives variance higher than {2} and higher costs of thinking in comparison with higher strategies.

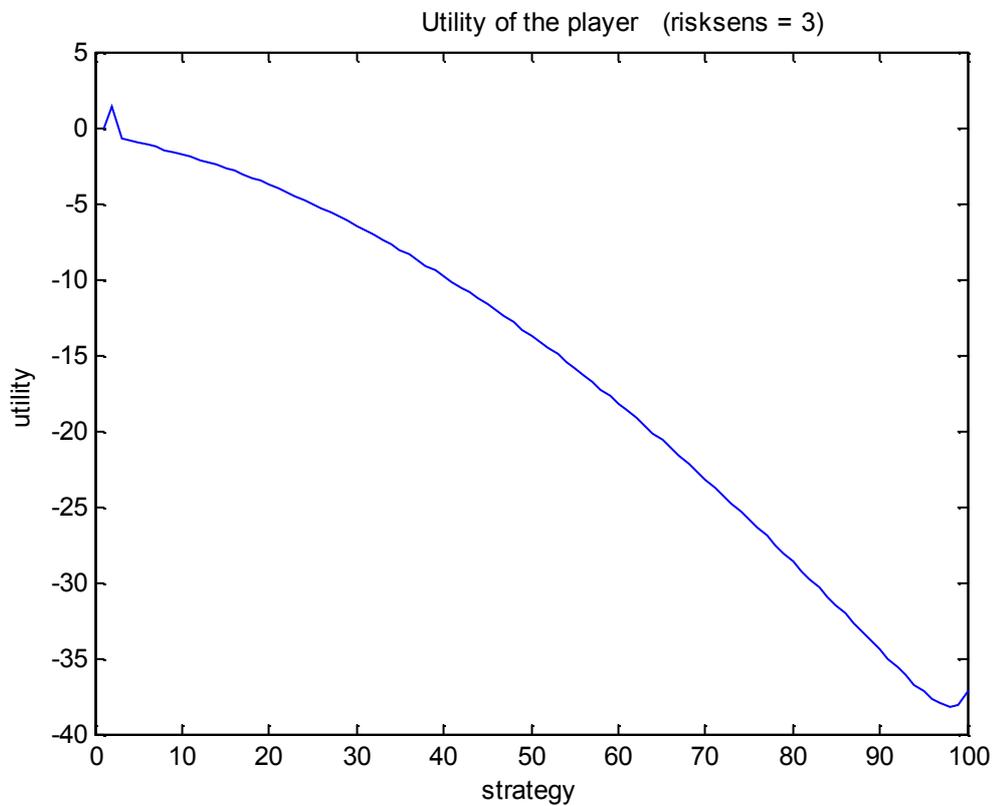
Global maximum in our model is definitely {100}, so those who played this strategy can be considered as a rational player, who maximizes their utility function.

Chart 6



Our model in general and risk sensitivity coefficient is consistent with all strategy played. For quite high coefficient of risk sensitivity (e.g. 3) agents would play Nash Equilibrium regardless to the distribution of beliefs (in case that they believe that there is some even small variation in counteragents strategies).

Chart 7



On other hand average risk sensitivity coefficients will lead to playing the average strategies, given this beliefs distribution (see charts 5.6 and 5.7). Using first approach of cost accumulation results will be skewed to the incentive to raise a strategy, because each further step of thinking decreases the utility in a same rate.

Chart 8

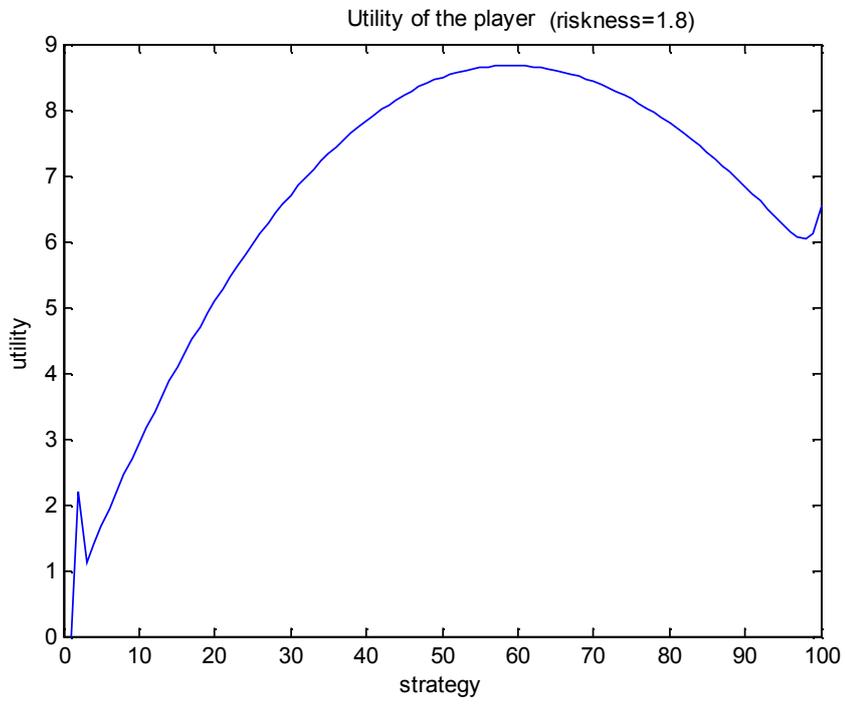
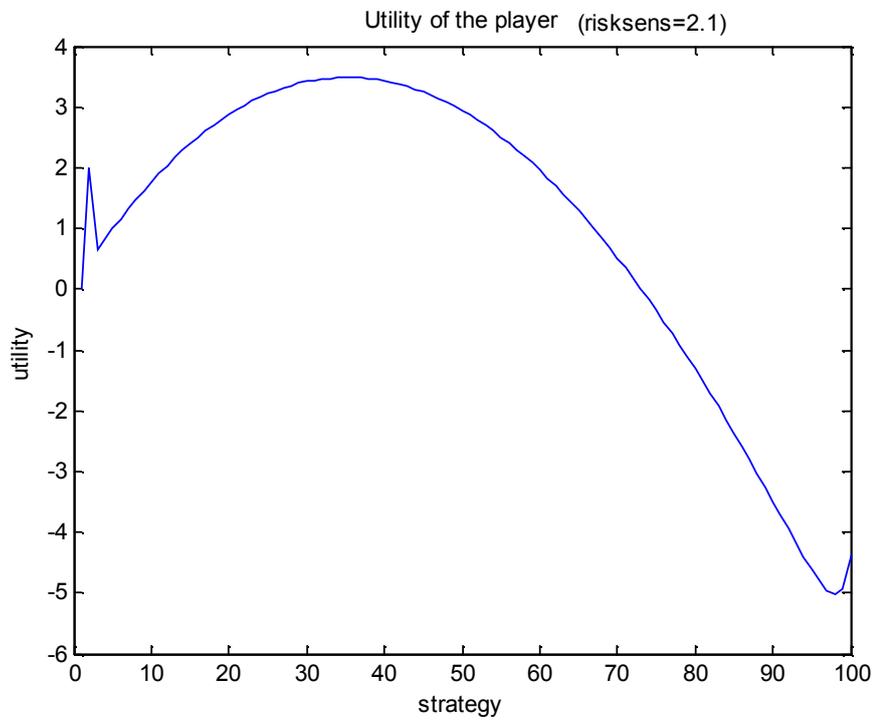


Chart 9



Other results and tables which describe the model are in Attachment C. These results show what strategy has each player in the experience, and given beliefs what was expected payoff, utility, standard deviation and cost of thinking.

Chapter 6

CONCLUSIONS

Given risk sensitivity coefficient greater than 1.377 even in absence of costs of thinking, but under aggregates beliefs of this experiment the strategy {99} becomes strictly dominated by strategy {100}. This fact shows that so-called “cooperative strategy” can be implemented if utility of agents contains not only expected payoff, but risk and risk sensitivity factor. Moreover it is just coincidence with cooperative strategy, because described model includes only “selfish” parameters, that is the utility function is not includes any kind of others’ utility. Hence, this result is showing that even without any cooperation considerations players can have an incentive to play so called “cooperative strategy”, as significant part of player do in reality. It is first such explanation of this behavior considering Travelers’ Dilemma.

On other hand average risk sensitivity coefficients will lead to playing the average strategies, given this beliefs distribution. However, if anyone believe that all counterparts will play NE, here will be no need in any risk sensitivity coefficient. Nevertheless, it can be presented even in such distributions, but it will not play the role. Hence, the classical Nash outcome here can be considered as a particular case, of this utility function. So, this approach provides explanation of playing any strategy (as data shows), and expand the Nash approach to the fit the reasoning in Travelers’ Dilemma.

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Aristotle,3

