MULTI-PERIOD STOCK MARKET
VOLATILITY FORECASTING:
EVIDENCE FROM EMERGING
MARKETS

by

Ovsiannykov Grygorii

A thesis submitted in partial fulfillment of
the requirements for the degree of

MA in Economics

Kyiv School of Economics

2011

Thesis Supervisor: Professor [Olesia Verchenko]

Approved by ______________________________
Head of the KSE Defense Committee, Professor Wolfram Schrettl

____

____

____

Date ______________________

Kyiv School of Economics

Abstract

MULTI-PERIOD STOCK MARKET
VOLATILITY FORECASTING:
EVIDENCE FROM EMERGING
MARKETS

by Ovsiannykov Grygorii

Thesis Supervisor: Professor Olesia Verchenko

This study compares the relative performance of direct, indirect and MIDAS volatility forecasting approaches to the widely used scaling approach based on the out-of-sample Mean Squared Forecasting Errors. Particularly, it investigates a robustness of MIDAS methodology on more volatile markets. Given lack of theoretical justification, the study carries out using empirical data for Poland, Ukraine and Russia stock indexes from 2002 to 2011. Based on GARCH(1,1) and linear MIDAS methodology quite unexpected conclusion that scaling method outperforms other more sophisticated models for weekly, bi-weekly and monthly horizon in terms of higher volatility predictive ability can be drawn. Finally, there is evidence that linear MIDAS perform better than direct and indirect methods for less volatile environment.

To my grandpa

TABLE OF CONTENTS

ACKNOWLEDGMENTS

The author wishes to express his sincere gratitude to his thesis advisor, Professor Olesia Verchenko, for her encouragement and guidance in the process of working on this research. Her patience and kindness allowed me to finish this thesis.

I want to thank my lovely Kate for her support. I would never do what I did without her. We are a great team!

My special thanks to all KSE professors for their help over this two years. I believe that I have learned some lessons from you that I will remember for the rest of my life. Finally, I want to thank all friends for their enormous infusion over last two years.

Chapter 1

INTRODUCTION

There is a countless number of investment decisions made every day all over the globe. The analysts ranging from the local commercial banks` in Vanuatu to the multinational JP Morgan's aim to invest in some combination of assets to get a profit. However, how do the investors choose a portfolio that is worth to invest in?

The most fundamental and basic theories of optimal investment decisions, such as Modern Portfolio Theory (Markowitz, 1952), and CAPM (Sharpe, 1964), suggest concentrating on the relationship between risk and return. Volatility of the price change for the underlined asset is considered as the main risk indicator. The adoption of the Basel Accords (1995, 2005) has increased the significance of obtaining precise measures of the volatility for the risk management purposes. Furthermore, modern option pricing theory (Black and Scholes, 1973) places volatility among the central elements of the model that determine the fair price for an option, or any other derivative instrument. Finally, accurate variance forecast is needed for public policy decisions, portfolio allocation, and developing rational hedging strategies.

Even though the importance of stock market volatility is apparent, little was done on this field for the emerging market economies. Although, the rapid growth of developing market economies made them desirable to the investors, this field of financial economics received less attention among academicians. It seems even more surprising, since the risk associated with the developing markets is considered to be higher than that for the developed countries. Consequently, due to the fact that the methods employed by practitioners are still quite rudimentary, it yields to even higher riskiness of the developing markets.

My research contributes to the field of multi-period ahead stock market volatility forecasting. Even though the most of the financial decisions done at least for two-three days horizons, the existing literature mainly focuses on the problem of one-day predictions. Therefore, scrutinizing the cases of one week volatility forecast and longer, allows developing more precise picture of the risk associated with the portfolio.

Over the last two decades countless number of papers attempted to propose the volatility forecasting approach that will give reliable results for horizons longer than several days ahead. Even though there was some substantial progress in this field, the ability of models to predict stock market volatility for longer horizons is still disputed.

Consequently, I believe that this research will add to the existing literature by analyzing and comparing the most frequently used multi-period volatility forecasting approaches. Along with a simple scaling method, direct method (GARCH), indirect method, and mixed-data sampling (MIDAS) models will be considered.

The scaling method, also known as a $\sqrt{k}$-scaling rule, is the most primitive way to produce multi-period volatility forecasts. It consists of scaling the one-period-ahead forecasts by the horizon of interest. Specifically, a k-days horizon implies a scaling factor of $\sqrt{k}$.

Even though this method is widely used among practitioners due to popular Riskmetrics approach developed by JPMorgan/Reuters (1996) and the suggestions in the Basel II agreement, several alternative approaches exist. The first alternative is to estimate a GARCH model and then use it to form direct predictions of the variance. This approach is referred to as the direct method.

The second alternative is referred to as an indirect method and was proposed by Marcellino, Stock, and Watson (2006). It consists of estimating daily autoregressive volatility forecasting model and then iterating over the daily forecasts for the required number of periods to obtain multi-period forecast of volatility. Finally, there is a relatively new mixed-data sampling (MIDAS) methodology proposed by Ghysels, Santa-Clara, and Valkanov (2005, 2006). According to these authors, a MIDAS approach "uses daily squared returns to produce directly multi-period volatility forecasts and can be viewed as a middle ground between the direct and the iterated approaches".

Comparison of different models will be done using the out-of-sample mean squared forecast error (MSFE) procedure. Furthermore, a conventional test of model forecasting ability, like those suggested by Giacomini and White (2006), will be conducted in order to investigate the difference in the predictive ability of these methods.

My study is based on publicly available data for stock market indexes in Ukraine, Russia, and Poland from 2002 to 2011. I will use daily market data to calculate daily, weekly, bi-weekly, and monthly stock market portfolio returns.

The main question of this research is which of the alternative approaches (direct, indirect and MIDAS) have the highest predictive ability on these emerging markets. Particularly, the study investigates how robust MIDAS methodology, which was found to be the most effective for the US market, is in more volatile markets.

The study will contribute to the existing literature in several ways.

Firstly, it addresses the problem of volatility forecasting for Ukrainian, Russian and Polish stock markets. While these markets are known to be more volatile and less liquid than leading emerging markets, like South Korea, China etc., to my best knowledge the existing literature does not cover these markets. *Secondly*, the study investigates whether long-horizon forecasts of the volatility of Ukrainian, Russian and Polish market returns produced by these methods are more accurate than widely used scaling up approach. By studying this question, the conclusion about the multi-period volatility predictability at the specified markets can be drawn. Even though it might seem apparent that given high predictability of one-period ahead volatility, one could expect reliable results for multi-period case, yet as evidenced by Christoffersen and Diebold (2000) the opposite is observed. The reason for this as indicated by Ghysels et all (2009) is that the model and parameter uncertainty as well as model instability result in unpredictability. *Thirdly*, the study investigates the relative performance of the different models, which showed to yield reliable results for the US market, on the emerging market case. One could expect different results because of the different nature of emerging markets compared to developed markets. Because of the shortage of theoretical ground in this field, there is a possibility that the result of comparison between models could be data driven. However, the results of Ghysels et all (2009) were quite sharp so that the conclusion about the best method for the US market could be clearly drawn. Taking the latter into account, the conclusion about the ability to find the method that performs better for the emerging markets will be made based on our results.

In the next section I will analyze the main literature. Next, I will describe the alternative models, discuss and data, and present empirical results. In the final chapter I will make conclusions about the performance of alternative forecasting models.

C h a p t e r   2

LITERATURE REVIEW

Volatility of the stock market returns is considered to be one the most important variables for financial economics. To large extent, the practitioners usually make their financial decisions for the relatively long horizons, which is one week and longer. For instance, due to the fact that reallocation of the portfolio is costly, the trader would rather decide to hold some stocks for a few days than sell them in response to a price changes. Therefore, an efficient method for obtaining long horizon stock price volatility should be developed.

Even though the importance of the multi-period ahead volatility forecasts is obvious, the existing research mainly concentrated on the question of forecasting one-day stock prices volatility. There are numerous papers in this field, however, Engle (1982), Bollerslev (1986) and Hansen and Lunde (2005) are considered as the most influential. Although, one-period volatility measure has limited applications in the real-world situation, these fundamental studies gave rise to further research.

Despite the fact that the framework for the one-period ahead volatility forecasting was introduced several decades ago, there is still no unanimity in the field with respect to the problem of multi-period ahead stock volatility forecasting. Even five years ago there was the erroneous belief among the researchers that stock market return is not forecastable. Meanwhile, a number of studies compared existing methodologies trying to obtain precise estimates for the horizons longer than ten days.

Particularly, West and Cho (1995) compared relative forecasting performance for four models: nonparametric, autoregressive, GARCH and univariate homoskedastic models. They used weekly exchange rates for the USD market to compare out-of-sample realization of the changes in the exchange rates with the estimates predicted by the scrutinized models. As a result they found that GARCH performs better only for a one week horizon. However, all the analyzed models fail to pass benchmark test of forecast efficiency for two weeks and longer: the null that all models have the same mean squared forecast error (MSFE) cannot be rejected. Since the MSFE criterion is a commonly accepted measure of forecast efficiency in the literature, it will also be used in the present research to compare the predictive power of alternative models. The fact that a GARCH model showed more precise results suggests including it into the set of scrutinized models for the case of emerging markets.

Christoffersen and Diebold (2000) focused on the global foreign exchange and bond markets. They introduced model-free procedure of comparing volatility forecasts across different horizons. However, their results were in consistency with the previous papers in the sense that they also found no trustworthy results beyond ten days horizon.

Taking into account relative difficulties in developing worthy model for predicting stock price volatility over the long-horizon, the practitioners frequently use a primitive scaling rule. It was proposed by JPMorgan/Reuters (1996) and is called the Riskmetrics approach. It was largely adopted by the practitioners due to its relative easiness. Given its wide use, this rule will be used as the simplest possible benchmark to be compared against more sophisticated procedures.

Marcellino et al (2006) proposed to estimate a daily autoregressive volatility model for obtaining volatility predictions. The model is then iterated forward for the required number of periods to produce as volatility forecast. Similarly to the present study, these authors compared it with the benchmark GARCH model. The comparison was done by means of MSFE and parametric bootstrap method. The authors concluded that the so-called "iterated" model outperforms the conventional GARCH model, which also suggests using it for the case of emerging markets. Finally, they noticed that iterated model gives better volatility estimates when direct method than prediction horizon increases.

Ghysels et all (2009) used the MIDAS procedure and compared it with the iterated and direct approaches. The researchers concluded that the direct model gives the worse results; however, iterated approach is suitable for up to 10 days horizon. Moreover, MIDAS methodology appeared to be by far the most efficient method for stock market volatility forecasting for 10-days and longer. This fact suggests estimation of the stock market volatility model based on the MIDAS approach as well. Finally, the authors performed formal tests, such as West (1996) and Giacomini and White (2006), to show that the null that MSFE is equal for three approaches is rejected at the conventional level of significance.

Even though numerous studies compare different volatility forecasting models, this research is different from other studies for a couple of reasons. Firstly, unlike other authors, I will focus on emerging markets that are known to be more volatile than other markets. Secondly, the data set consists of stock market indexes, while Marcellino et al. (2006), for instance, worked with the US macroeconomic time series. Thirdly, I will expand comparison of volatility forecasting procedures for stock markets by adding MIDAS approach that showed to be efficient. Therefore, building on recent works by Marcellino et all (2006) and Ghysels et al (2009), I will identify the model that gives the most accurate predictions for the emerging stock market volatility.

Switching our attention to the existing literature related to the problem of stock market volatility prediction for the emerging markets, one may find that there is a limited number of papers concerning multi-period horizon volatility forecasting. Specifically, Gokcan (2000) found that even though stock market return series display skewed distribution, so theoretically non-linear EGARCH is more appropriate, for emerging markets GARCH(1,1) model performs better. Moreover, Pryymachenko (2003) estimated Ukrainian stock price volatility using TGARCH. Liu et all (2009) verified that using China`s stock prices for all forecasting horizons GARCH-SGED shows much better volatility predictive ability to GARCH-N model.

Finally, Alper et all (2008) compared relative performance of conventional GARCH(1,1) and a linear univariate MIDAS models for developed and emerging economies. They used weekly stock market data for estimation volatility forecasting performance of ten developing and four developed markets. The results do not look as impressive as it was expected for both cases. Based on MSFE MIDAS outperforms GARCH for all markets except Argentina`s market (MERVAL) and based on West (2006) the MIDAS showed significant difference only for four emerging economies.

There are several implications of the aforementioned results. Firstly, different sampling frequency should be taken, since MIDAS approach is showed to be more efficient for daily data (Ghysels et al 2009). Also, as suggested by Marcellino et all (2006) iterated approach should be also scrutinized, as it showed to produce more accurate results over benchmark GARCH for some developed markets.

Chapter 3

METHODOLOGY

Before describing the models for conditional variance, I would like to refer to Bollerslev et al. (1992) and Granger and Poon (2003) for a survey of modern literature on conditional volatility. These studies scrutinize the volatility models that showed to outperform alternative models in different studies.

Henceforward, I will follow Ghysels et al (2009) and use the following notation. Daily stock prices will be denoted as *d* where d=1, 2, …, D and returns for *k*-days horizons will be denoted by t = 1, 2, …, T_k, where T_k=[D/k]. Let

$$r_t = (\log P_{d+1} - \log P_d)\,100 \qquad (1)$$

be daily returns, where P_d is the stock price at period d. Then,

$$R_t^k = (\log P_{d+k} - \log P_d)100 \qquad (2)$$

will be denoting continuously compounded returns for the *k*-period. Further, let me denote the information set of daily returns in period *t* by $I_t = \{r_0, r_1, \ldots, r_{t-1}, r_t\}$. Similarly, the information set of the *k*-period, continuously compounded returns will be denoted by $I_t^k = \{R_0^k, R_1^k, \ldots, R_{t-1}^k, R_t^k\}$.

Further, the volatility forecasts using different methods will be denoted by $V_c = (a, k, i)$. Where c denotes the forecasting approach: either direct method (d), iterated (i) and MIDAS (m), a is the starting point of forecast, k is the forecast horizon and i is the information set used. For instance, $V_i = (t, 1_k, I_t)$ denotes conditional forecasts of the volatility *k*-days ahead at time *t* using both the informational set I_t and iterated method of forecasting.

Scaling Volatility Forecasts

Let me start description of the methods of predicting conditional variance from the widely used scaling method. Generally, the algorithm is following. First, a model is estimated using daily variance of returns up to period *t*. In this study it was done using standard GARCH$(1,1)$ approach, based on the relative AIC and BIC information criteria. Second, the forecast of the variance is obtained for the next period, *t+1*. Third, daily variance for *t+1* is converted to a weekly, bi-weekly, and monthly volatility return by multiplying by $\sqrt{k}$, where k is the length of horizon of interest. For instance, in order to obtain weekly volatility forecast using scaling method, each Monday forecast is multiplied by $\sqrt{5}$.

Even though Diebold et al. (1998) indicate that this approach is spurious as volatility change is magnified with longer horizon, it is widely used among practitioners since it is recommended by Basel II agreement. Consequently, scaling method is used as a benchmark for comparing the performance of other models on volatile markets.

Direct Volatility Forecasts

The direct method appears to be the most widely used for volatility forecasting purposes in the academic papers. It consists of estimating the multi-horizon volatility directly as one step ahead using *k*-period returns R_t^k. Using findings from the relevant papers that studied problems of the direct volatility forecasting on the emerging markets, $V_D = (T, 1_k, I_t^k)$ is modeled as a general GARCH(p,q).

In order to choose the appropriate p and q, standard information criteria are used such as Akaike Information Criterion (AIC) and Bayes Information Criterion (BIC). For this purpose, different GARCH specifications up to $p = q = 3$ are estimated using the whole sample for each horizon k. Then, AIC and BIC are computed for each model specification for each horizon of interest. Finally, an appropriate model specification is chosen based on the lowest value of each information criteria.

As suggested by Gokcan (2000) AR(1)-GARCH$(1,1)$ model has high predictive ability on the emerging markets despite the fact that market returns have asymmetric distribution, in which case the EGARCH would seem to be more appropriate. Therefore, let me briefly describe forecasting algorithm based on GARCH$(1,1)$.

The standard econometric specification for GARCH$(1,1)$ is constructed as:

$$R_t^k = \alpha_1 + \alpha_2 R_{t-1}^k + \varepsilon_t,\ \varepsilon_t = \sigma_t z_t, \text{ and } z_t | I_t^k \sim f(0,1) \quad \textbf{(3)}$$

where α_1 and α_2 are constant parameters, ε_t is innovation process and $f(0,1)$ is a density function with zero mean and unit variance. Moreover, the equation for conditional variance is:

$$\sigma_t^2 = \rho_1 + \rho_2 \varepsilon_{t-1}^2 + \rho_3 \sigma_{t-1}^2 \quad \textbf{(4)}$$

where ρ_1, ρ_2, and ρ_3 are non-negative parameters. Furthermore, an additional restriction $\rho_2 + \rho_3 < 1$ should be imposed to ensure stationary positive variance. In order to proceed with the estimation of the GARCH model, the assumption about distribution of the error term, z_t, should be made. Based on the relevant literature in this field, particularly Liu et all (2009), standard normal distribution (GARCH-N) is an appropriate for the error term.

The variance forecast is made in the following way. Firstly, k-period conditional variance up to period t is estimated using k-period returns by equation (4). Then, forecast of σ_{t+k}^2 conditional on I_t^k is made as:

$$E\left(\hat{\sigma}_{t+k}^{2}\middle|I_{t}^{k}\right) = \hat{\rho}_{1}^{k} + \left(\hat{\rho}_{2}^{k} + \hat{\rho}_{3}^{k}\right)\left(\sigma_{t}^{2}\,|I_{t}^{k}\right) \qquad (5)$$

This approach delivers reliable results since it does not display a bias. Moreover, as suggested by Hansen and Lunde (2005) given the parsimonious nature of the GARCH method one may expect robust estimates of the stock prices volatility.

Indirect Volatility Forecasts

In addition to the previous two approaches, volatility forecasts, $V_t = (T, 1_k, I_t)$, based on the daily returns are estimated indirectly. The indirect method implies that daily forecasts are iterated for the necessary number of periods. At the first stage, the daily autoregressive volatility forecasting model is estimated using AR(1)-GARCH$(1,1)$ as described in the previous section. Then, the forecast of $\hat{\sigma}_{t+h}^{2}$ are constructed as:

$$E(\sigma_{t+h}^{2}\|I_{t}^{k}) = \hat{\rho}_{1} + (\hat{\rho}_{2} + \hat{\rho}_{3})E(\sigma_{t+h-1}^{2}\|I_{t}) \qquad (6)$$

where $E\left(\sigma_{j}^{2}\middle|\Omega_{t}\right) = \sigma_{j}^{2}$ for $j \leq t$ in order to obtain the iterated volatility for the *k*-periods ahead.

Mixed Data Sampling Volatility Forecast

Finally, the performance of the most recent Mixed Data Sampling methodology is studied. It consists of usage the informational set I_t in order to estimate variance forecast $E\left(\hat{\sigma}_{t+k}^{2}\middle|I_{t}^{k}\right)$ directly.

The simple general univariate MIDAS has the following econometric specification:

$$Y_{t}^{k} = \mu_{k} + \varphi_{k}\sum_{j=0}^{jmax} B(j,\theta)\, X_{t-j} + \varepsilon_{t} \qquad (7)$$

where $B(j,\theta)$ is a polynomial weighting function parameterized by a low-dimensional parameter vector θ and X_t is the explanatory variable sampled *k* times faster than Y_{t}^{k}.

For studying research questions discussed above, the following MIDAS specification is used:

$$V_{t+1}^{k} = \mu_k + \varphi_k \sum_{j=0}^{jmax} B(j,\theta)\,[r_{t-j}]^2 + \varepsilon_t \qquad \textbf{(8)}$$

where V_{t+1}^{k} is the variance forecast such as $V_{t+1}^{k} = RV_{t+1}^{k} \equiv \sum_{j=1}^{k} r_{t+j}^{2}$ and the parameters of the equation above: intercept μ_k, slope coefficient φ_k and weighting scheme θ is estimated with quasi-maximum likelihood estimator. This model specification has several appealing features of MIDAS approach. On the one hand, it parsimoniously parameterizes $B(j,\theta)$ so that the weights associated with the squared returns on day $t-j$ are estimated in-sample and used to produce out-of-sample forecast. On the other hand, this method incorporates data sampled at different frequencies and, therefore, might bring gains in efficiency compared to direct and indirect approaches.

Several different specifications of the weighting function can be used to predict future volatility. Even though Alper et all (2008) suggest that Beta lag polynomial outperform other specifications, taking into account sample driven nature of the performed analysis, the study will expand previous research for the emerging markets and investigate linear scheme. Following the study of Ghysels et all (2009) performed for the US stock market, linear function specification is described as:

$$B(j) = \frac{1}{jmax} \qquad \textbf{(9)}$$

This weighting scheme places the same weight on all lagged returns and is appealing in the sense that there is no need to estimate parameter vector θ.

Linear MIDAS specification has three parameters to be estimated using quasi- maximum likelihood estimator. The estimation procedure is started from writing likelihood function in general form:

$$\varepsilon_t = V_{t+1} - \mu - \varphi \sum_{j=0}^{jmax} b(j,\theta) r_{t-j}^{2} \qquad \textbf{(10)}$$

$$L = \prod_{t=1}^{\tilde{T}} f(\varepsilon_t) \quad (11)$$

where $\tilde{T} = T - j_{max}$ and $f = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\varepsilon^2}{2\sigma^2}}$.

Taking logarithms from both part yields:

$$\log L = \sum_{t=1}^{\tilde{T}} [- \ln \sigma - \frac{(V_{t+1} - \mu - \varphi \sum_{j=0}^{jmax} b(j,\theta) r_{t-j}^2)^2}{2\sigma^2}] \quad (12)$$

Then taking FOC with respect to the unknown parameters σ, μ, φ, yield the following three equations in three unknowns:

$$-\sigma^2 \tilde{T} + \sum_{t=1}^{\tilde{T}} [V_{t+1} - \mu - \varphi \sum_{j=0}^{jmax} b(j,\theta) r_{t-j}^2]^2 = 0 \quad (13)$$

$$\sum_{t=1}^{\tilde{T}} V_{t+1} - \mu \tilde{T} - \varphi \sum_{t=1}^{\tilde{T}} \sum_{j=0}^{jmax} b(j,\theta) r_{t-j}^2 = 0 \quad (14)$$

$$\sum_{t=1}^{\tilde{T}} V_{t+1} \sum_{j=0}^{jmax} b(j,\theta) r_{t-j}^2 - \mu \sum_{t=1}^{\tilde{T}} \sum_{j=0}^{jmax} b(j,\theta) r_{t-j}^2 - \varphi \sum_{t=1}^{\tilde{T}} \sum_{j=0}^{jmax} b(j,\theta) r_{t-j}^2 \sum_{i=0}^{jmax} b(i,\theta) r_{t-i}^2 = 0 \quad (15)$$

Solving these three equations gives the optimal $\sigma^*, \mu^*, \varphi^*$ that maximize log likelihood function. Moreover, the described framework is general for any weighting scheme specification.

Comparing the Forecasts

The standard out-of-sample forecast error is computing as the following:

$$e_{F,T+1}^k = V_p(T, 1_k) - V_F(T, 1_k) \quad (16)$$

where $V_p(T, 1_k)$ is the population conditional volatility *k*-period ahead given past daily data and $V_F(T, 1_k)$ is the volatility forecast estimated either by scaling, direct, indirect, or MIDAS method. However, since the true forecast is unknown, the proxy should be used.

The out-of-sample volatility forecast error is computed based on the known volatility:

$$u_{F,T+1}^k = RV_{T+1}^k - V_F(T, 1_k) \quad (17)$$

where realized volatility RV^k_{T+1} serves as a proxy for the unknown true volatility V_p and $RV^k_{t+1} \equiv \sum_{j=1}^{k} r^2_{t+j}$. Moreover, the sample MSFE is:

$$MSFE^k_F = \frac{1}{T_2 - T_1 + 1} \sum_{t=T_1}^{T_2} \left(u^k_{F,t}\right)^2 \qquad \textbf{(18)}$$

Based on the relative MSFE each forecasting method would be compared with respect to the time horizon k. Moreover, as showed by Patton (2007) ranking based on the known $u^k_{F,T+1}$ will be the same as when unknown $e^k_{F,T+1}$. Therefore, MSFE preferred to the other forecasting evaluation methods.

Tests for the Predictive Ability

In order to gauge whether different forecasting procedures produce statistically different results a benchmark Giacomini and White (2006) test is used. It is widely used in the literature and particularly suggested by Ghysels et all (2009) as relevant.

Giacomini and White (2006) formulated null hypothesis and proposed the following test of conditional predictive ability:

$$H_0: E\left[\left(u^k_{F1,t}\right)^2 - \left(u^k_{F2,t}\right)^2 | I_{t-1}\right] = 0 \qquad \textbf{(19)}$$

where I_{t-1} is the information set available at period $t-1$. This test seems more appropriate than another popular West (1996) since the later procedure tends to sort out correctly specified model. Nevertheless, as suggested by Giacomini and White (2006), even misspecified model may produce better volatility forecast in case when the parameters are imprecisely estimated in the model that well approximates the data generating process. Produced values under Giacomini and White (2006) test has chi-square distribution with one degree of freedom, so that different forecasting approaches can be compared to critical value obtained from the table of chi-square distribution.

Chapter 4

DATA

In my study I focus on the stock markets of Ukraine, Poland and Russia as these markets seem to be under investigated in this research field. In particular, the data set consists of daily stock returns of PFTS (PFTS Ukraine Stock Exchange, Ukraine), MICEX (Moscow Interbank Currency Exchange, Russia), and WIG 20 (Warsaw Stock Exchange, Poland). The daily stock market indices are used to calculate log returns, using expression (1). In particular, closing values of market indexes are used for the period from November 29, 2002 to February 24, 2011 for WIG 20 and PFTS, and for the period from November 27, 2002 to February 24, 2011 for MICEX. This time period is chosen, since it captures the time over which all three indexes are available. The data for MICEX and PFTS are taken from the official web-sites of the respective exchanges, and historical data on WIG 20 is taken from a financial web-site (www.euroinvestor.co.uk)[1].

Tables 1 through Table 3 contain the descriptive statistics for the stock market returns for each forecasting horizon. In addition, summary statistics includes variance of squared returns. The reason beyond this is that squared returns are used in this study as a proxy for variance of the returns. This proxy is common in the literature.

It follows from the summary statistics that mean daily returns on PFTS is approximately twice higher than daily returns on MICEX and four times higher than daily returns on WIG 20. Contrary, the highest variance for daily returns is observed for the MICEX index. Moreover, the daily return series are not normally distributed as indicated by Jarque-Bera test, since the null that time series is normally distributed is rejected based on the given chi-square statistics.

[1] http://www.euroinvestor.co.uk/stock/historicaldata.aspx?id=639281

Using daily returns, *k*-period continuously compounded, non-overlapping returns are computed according to (2) for $k = 5$ (weekly), $k = 10$ (bi-weekly), and $k = 20$ (monthly).

Similarly to the conclusions for daily returns, the highest mean of weekly returns are on PFTS market, while variance are on MICEX. For the longer than weekly horizons PFTS market appears both to yield the higher return and to be the most volatile. Furthermore, all series are not normally distributed as the null cannot be rejected.

Finally, WIG 20 appears to have the lowest variability of the volatility across markets as indicated by variance of squared returns. On the other hand, MICEX has higher variability of volatility for daily and weekly horizon, whereas PFTS has the highest change of volatility in time for longer horizon. An analysis of variance of squared returns allows drawing a conclusion that is not only level of volatility is higher for PFTS, but also time varying volatility is also higher.

Chapter 5

EMPIRICAL RESULTS

Based on the discussed methodology four forecasting approaches are used to identify volatility predictions for k –periods ahead in the future.

Estimation of the volatility forecasting models based on scaling, direct and indirect approaches start from identifying an appropriate model specification. As discussed in the previous sections, volatility process is modeled as a general GARCH(p,q) that is consistent with the existing practices in this field. In order to identify the appropriate number of lags p and q, AIC and BIC information criteria are used for each market and for each horizon of interest. I compared AIC and BIC values for different GARCH(p,q) specifications for $p,q = \overline{1,3}$. Since the models under investigation are nested and the same sample size is used in order to compute values of information criteria, those numbers can be compared directly without any adjustments. Similarly to Gokcan (2000), GARCH$(1,1)$ seems to be the most appropriate, since almost for all horizons and markets, it produces the lowest values of AIC and BIC information criteria. Even though GARCH$(3,1)$ is the second best model in terms of given selection criteria, I decided to focus on a more parsimonious GARCH$(1,1)$ model.

The estimation start with predicting one-day ahead forecast. For this matter, three fourth of the daily stock market returns are used to obtain the estimated values of the coefficients in equations (3) and (4). For instance, first 1530 observations of daily stock returns for WIG20 are used to obtain the estimators of the daily volatility model. Then, this procedure is repeated for the next 1530 observations, beginning from the second. Consequently, we obtain a sample of 510 coefficient values. Similarly, *k*-period stock returns are used to obtain the values of the coefficients for *k*-period volatility model.

Further analysis differs for scaling and indirect approaches from the direct procedure. The reason for this is that the former approaches use daily volatility model in order to predict the volatility *k*-period ahead, while the latter relies on the *k*-period volatility model.

For scaling approach, I use one-day ahead volatility prediction and multiply it by $\sqrt{k}$, where *k* is the horizon of interest. Particularly, in order to obtain prediction for the November`s volatility using scaling approach, I first obtain one-period ahead forecast based on GARCH$(1,1)$ model for the first trading day of November, and then multiply it by $\sqrt{20}$.

Similarly to the scaling procedure, in order to get multi-period forecast using indirect procedure, one should start from one-day ahead volatility forecast estimated using GARCH$(1,1)$. Then, this prediction enters an equation for the periods *t+2*, *t+3*, and so on till period *t+k*, where *k* is horizon of interest. More generally, indirect forecasting proceed as in equation (6).

Contrary to the previous two approaches, direct forecast is obtained based on the estimates from weekly, bi-weekly, or monthly model. For instance, I start with regressing monthly returns based on GARCH$(1,1)$ to get the estimates of the coefficients, and then use the known monthly volatility at period *t*, σ_t, to predict the volatility for the next month.

In addition to previously mentioned models, I estimate Mixed-Data Sampling volatility forecasting model. Particularly, the weighting function is linear and is described by expression (9). In order to receive estimators of the regression coefficients matrix, log-likelihood procedure is employed. MIDAS estimation consist of two steps. First, optimal j_{max} is identified using both log likelihood

(LL) and Akaike Information Criteria (AIC). For this purpose, values of LL and AIC selection criteria are calculated based on the same sample size $\tilde{T} = T - j_{upper}$, where j_{upper} is the maximum value of j_{max} that was assumed to be forty, since the optimal number of lags is found to be much lower than forty, this bound is not restrictive. Almost for all markets and horizon both LL and AIC suggested the same maximum number of lags to be included into MIDAS specification. Moreover, the lowest number of lags were chosen where LL and AIC suggested different number of j_{max} to fit the most parsimonious model.

At the second step of MIDAS methodology, I estimate coefficient matrix again using the firsts two third of the entire sample. For instance, first 1535 observations is used to estimate μ and φ based on MICEX data and forecast for period *t+1* is obtained. Then, this procedure is repeated, so that the sample of 1535 observations, beginning from the second observation, is taken. As a result, I obtained another 510 forecasts of daily volatility, 102 forecasts of weekly volatility, 51 forecasts of bi-weekly and 25 forecasts of monthly volatility using MIDAS approach using scaling, direct and iterated methodology.

In Tables 4 through 6, I present MSFE associated with different models. The results are quite unexpected and contradict the previous findings in the literature. The main difference is that scaling approach appears to be the most efficient in terms of MSFE for all markets and almost for all horizons. Tables 4 through 6 indicate the most efficient approach in terms of MSFE given *k* in bold letters.

Moreover, linear MIDAS performs worse in terms of MSFE for two markets: it performs worse than direct approach for $k = 10, 20$ for MICEX and worse than indirect approach for $k = 20$ for PFTS. Furthermore, linear MIDAS appears to be constantly the second best model only for the Polish market, while it was beaten by direct and indirect approaches for the Russian and Ukrainian stock markets respectively.

To sort out the models that produce statistically different forecasts Giacomini and White (2006) test of predictive ability is used. It indicates that for all markets these four approaches have the same accuracy of daily volatility forecasts in statistical terms. However, when horizon increase it is possible to identify the best approach in terms of predictive ability.

The obtained results could be explained by a number of factors. On the one hand, low liquidity of the scrutinized markets could be the reason for these unconventional results. On the other hand, our data sample includes observations for crisis period between 2008 and 2010, when high market volatility can be explained by a panic among the investors. These are rather extreme market circumstances and traditional approaches might find it particularly difficult to fit the observed data.

However, the most straightforward explanation is that linear MIDAS, in fact, performs worse on the more volatile markets. That is partially consistent with finding of Alper et al (2008). Given both a shortage of theoretical ground and data driven character of the research question, I believe that analyzed linear weighting scheme has lower predictive power than other schemes.

Chapter 6

CONCLUSIONS

The study considers the performance of direct, indirect and linear MIDAS approaches at the volatile environment of the emerging markets. In addition, widely used scaling method is used. I compared these approaches based on the Mean Squared Forecasting Errors (MSFE) computed for different forecasting horizons. Since this research field lacks theoretical justification and general conclusions are hard to make, this research question is investigated based on the empirical data sample of daily stock prices from 2002 to 2011 for Polish, Russian and Ukrainian stock markets.

The results are somewhat unexpected as the scaling approach appears to outperform other methods and the difference in predictive ability is statistically significant. In addition, linear MIDAS that was of particular interest for this study appears to perform worse even than the second best model in three cases. Finally, there is evidence that linear MIDAS shows higher predictive ability for less volatile Polish market than for more volatile markets of Russia and Ukraine.

This study is of great interest for the investors in these markets, since we are able to identify a model of volatility forecasting that produces more robust results than other models. Further research would include analysis of different MIDAS specifications. Particularly, as suggested by Alper et all (2008), beta weighted scheme would be compared with linear scheme.

WORKS CITED

Alper, C. Emre, Salih Fendoglu, and Burak Saltoglu. 2008. Forecasting Stock Market Volatilities Using MIDAS Regressions: An Application to the Emerging Markets, MPRA Paper No. 7460, posted 05. March 2008.

Andersen, Torben, Tim Bollerslev, Frank X. Diebold, and Peter Labys. 2001. The distribution of exchange rate volatility, *Journal of American Statistical Association* 96, 42–55.

Black, Fischer, and Myron Scholes. 1973. The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* 81, 637-654.

Bollerslev, Tim. 1986. Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics* 31, 307–327.

Bollerslev, Tim, Ray Y. Chou, and Kenneth F. Kroner. 1992. ARCH modeling in finance: A review of the Theory and Empirical Evidence, *Journal of Econometrics* 52, 5-59.

Christoffersen, Peter, and Francis Diebold. 2000. How relevant is volatility forecasting for financial risk management?, *Review of Economics and Statistics* 82, 12–22.

Diebold, F.X., T. Schuermann, and J. Stroughair. 1998. Pitfalls and opportunities in the use of extreme value theory in risk management, Wharton Financial Institutions Center, Working Paper 98-10.

Engle, R.F. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica* 50, 987–1008.

Ghysels, Eric, Antonio Rubia, and Rossen Valkanov. 2009. Multi-Period Forecast of Volatility: Direct, Iterated, and Mixed-Data Approaches, EFA 2009 Bergen Meetings Paper.

Ghysels, Eric, Pedro Santa-Clara, and Rossen Valkanov. 2005. There is a risk-return tradeoff after all, *Journal of Financial Economics* 76, 509–548.

Ghysels, Eric, Pedro Santa-Clara, and Rossen Valkanov. 2006. Predicting volatility: getting the most out of return data sampled at different frequencies, *Journal of Econometrics* 131, 59–95.

Giacomini, Raffaella, and Halbert White. 2006. Tests of conditional predictive ability, *Econometrica* 15, 549–582.

Gokcan, Suleyman. 2000. Forecasting volatility of emerging stock markets: linear versus non-linear GARCH models, *Journal of Forecasting, Volume* 19, Issue 6, 499–504.

Granger, C. W. J, and S.-H. Poon. 2003. Forecasting Volatility in Financial Markets: *A Review. Journal of Economic Literature* 41(2): 478-539.

Hansen, Peter, and Asger Lunde. 2005. A forecast comparison of volatility models: Does anything beat a GARCH(1,1)?, *Journal of Applied Econometrics* 20, 873–889.

J.P.Morgan/Reuters. 1996. Riskmetrics – technical document, J.P. Morgan/Reuters, New York.

Liu, Hung-Chun, Yen-Hsien Lee, and Ming-Chih Lee. 2009. Forecasting China Stock Markets Volatility via GARCH Models Under Skewed-GED Distribution, *Journal of Money, Investment and Banking*, issue 7.

Marcellino, Massimiliano, James Stock, and Mark Watson. 2006. A comparison of direct and iterated multistep AR methods for forecasting macroeconomic time series, *Journal of Econometrics* 135, 499–526.

Patton, Andrew. 2007. Volatility forecast comparison using imperfect volatility proxies, Working Paper, London School of Economics.

Pryymachenko, Iryna. 2003. Stock price volatility and its determinants on the emerging financial market of Ukraine, MA thesis, Economics Education and Research Consortium.

West, Kenneth and D. Cho. 1995. The predictive ability of several models of exchange rate volatility, *Journal of Econometrics* 69, 367–391.

West, Kenneth. 1996. Asymptotic inference about predictive ability, *Econometrica* 64, 1067–1084.

APPENDIX

Table 1. WIG 20 market returns (November 29, 2002 till February 24, 2011)

	WIG 20			
	$k = 1$	$k = 5$	$k = 10$	$k = 20$
Number of observations	2040	408	204	102
Mean of returns	0.039	0.193	0.385	0.578
Variance of returns	2.676	14.472	24.358	40.520
Variance of squared returns	32.247	807.713	1787.135	4523.439
Skewness	-0.221*	-0.390*	-0.589*	-0.654*
Kurtosis	5.525*	4.935*	4.187*	3.980*
JB test	5.e-122*	8.7e-17*	6.9e-06*	5.2e-04*

* Statistically insignificant at the 10% level

Table 2. MICEX market returns (November 27, 2002 to February 24, 2011)

	MICEX*			
	$k = 1$	$k = 5$	$k = 10$	$k = 20$
Number of observations	2045	409	204	102
Mean of returns	0.082	0.412	0.839	1.259
Variance of returns	6.171	28.124	51.151	81.999
Variance of squared returns	629.732	10094.1	16958.74	54290.62
Skewness	-0.177**	-0.0004**	-1.199**	-1.724**
Kurtosis	17.565**	13.769**	8.022**	10.019**
JB test	0**	0**	6.7e-58**	5.6e-76**

* MICEX daily returns are calculated for the period November 27, 2002 to February, 2011

** Statistically insignificant at the 10% level

Table 3. PFTS market returns (November 29, 2002 till February 24, 2011)

	PFTS			
	$k = 1$	$k = 5$	$k = 10$	$k = 20$
Number of observations	2025	405	202	102
Mean of returns	0.150	0.749	1.496	2.268
Variance of returns	4.560	27.243	68.714	131.70
Variance of squared returns	276.988	4853.194	27099.03	96968.63
Skewness	0.044*	-0.553*	-0.505*	-0.772*
Kurtosis	14.293*	7.790*	7.003*	7.088*
JB test	0**	2.7e-89*	7.1e-32*	7.2e-24*

* Statistically insignificant at the 10% level

Table 4. Multi-Period MSFEs of Volatility Forecasts - WIG 20

	Horizon			
	$k = 1$	$k = 5$	$k = 10$	$k = 20$
Linear MIDAS	1,4398*	0,5835*	0,4515(**)	0,2689
Scaling method	1,4395*	**0,4505**	0,2679*	**0,1612**
Direct method	1,4395*	0,5442*	0,3337(*) (**)	0,4305
Indirect method	1,4395*	1,0580	1,4200	1,6055

* Equal forecasting ability based on GW test under 5% confidence interval

** Equal forecasting ability based on GW test under 10% confidence interval

Table 5. Multi-Period MSFEs of Volatility Forecasts - MICEX

	Horizon			
	$k=1$	$k=5$	$k=10$	$k=20$
Linear MIDAS	2,4056*	0,9037*	1,5841	1,4014
Scaling method	2,3874*	**0,6678**	**0,6003**	**0,3609**
Direct method	2,3874*	0,8019*	0,9600	1,0832
Indirect method	2,3874*	1,7482	1,8670	2,0454

* Equal forecasting ability based on GW test under 5% confidence interval

Table 6. Multi-Period MSFEs of Volatility Forecasts - PFTS

	Horizon			
	$k = 1$	$k = 5$	$k = 10$	$k = 20$
Linear MIDAS	2,4083*	1,6750*	1,2719*	2,8810*
Scaling method	2,4649*	**1,3820**	1,0602*	**0,8734**
Direct method	2,4649*	2,7909	4,1957	2,5870*
Indirect method	2,4649*	1,6969*	2,1232	1,3243

* Equal forecasting ability based on GW test under 5% confidence interval